

PHOTONICS Research

Measurement-based building-block behavioral modeling of passive photonic circuits from magnitude-only responses

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Accurate frequency-domain characterization is essential for reliable modeling of photonic integrated circuits (PICs). In practice, PIC measurements are often performed using a tunable laser and an optical power meter, providing only magnitude responses and no phase information, which hinders the construction of physically consistent frequency- and time-domain models. This paper presents a robust approach that directly constructs rational models from magnitude-only frequency responses, ensuring that the resulting models share the same magnitude response as the PIC under study. Unlike existing methods based on magnitude-squared fitting and spectral factorization, which are frequently degraded by imaginary-axis zeros, the proposed method synthesizes a minimum-phase realization by explicitly handling these zeros. The synthesized phase is not the true physical phase; however, it is consistent with the Kramers–Kronig relations corresponding to the given magnitude response, thereby enabling the direct use of standard *S*-parameter approximation techniques to generate rational behavioral models. The resulting models can be applied to modulated signal transmission simulations in wavelength-division multiplexed (WDM) telecommunication scenarios. © 2026 Chinese Laser Press

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1. INTRODUCTION

Over the past decade, photonic integrated circuits (PICs), especially those fabricated using the silicon-on-insulator (SOI) platform, have experienced rapid advances in integration scale and complexity, driven by increasingly mature fabrication technologies [1–4]. These circuits are now widely used in high-speed optical interconnects, neuromorphic computing, quantum computing, and LiDAR, capitalizing on the speed and bandwidth advantages of optical signals. However, characterizing PICs often relies heavily on intensity-based or magnitude-only measurements, from which it is difficult to extract reliable phase information. Phase extraction typically requires complex coherent detection, interferometric configurations, or extensive calibration procedures, which are challenging in highly integrated on-chip environments. On top of that, it is often very difficult to de-embed the phase response of a building block from the response of the incoupling and out-coupling structures and connection waveguides. As a result, magnitude (intensity) response data are far more readily available than full complex (magnitude and phase) data, and this creates a significant barrier to obtaining accurate models for circuit-level simulation and optimization.

This is particularly true for passive PICs and building blocks, such as directional couplers, grating couplers, multi-mode interferometers, arrayed waveguide gratings, and various wavelength-selective filters. They are commonly characterized using a tunable laser source in combination with a power meter, as shown in Fig. 1, which provides only magnitude information over a specified wavelength range. Although optical vector analyzers (OVAs) are capable of capturing both magnitude and phase responses of PICs, achieving accurate and low-noise phase measurements requires stringent calibration, environmental control, and coherent reference signals [5–7]. These requirements significantly increase the sensitivity and operational complexity of such systems. Moreover, the high cost of OVAs poses a substantial barrier to their adoption, particularly for academic laboratories and smaller research groups. In addition, even when an OVA is available, accurate de-embedding remains a nontrivial task, further complicating practical phase measurements.

A similar challenge arises in the modeling of RF and microwave circuits, where phase information may be unavailable or unreliable. This situation commonly occurs in high-power microwave systems, in which accurate phase measurements

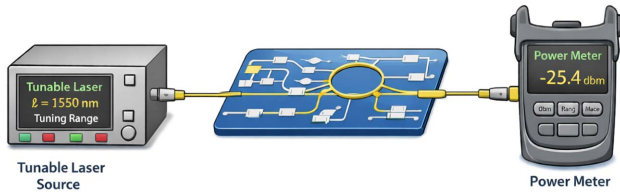


Fig. 1. Typical measurement setup for PICs using a tunable laser and an optical power meter, which provides magnitude-only frequency response data.

become increasingly difficult at high power levels or frequencies. To address this problem, the magnitude vector fitting (MVF) method was developed [8,9]. It first constructs a rational approximation of the magnitude-squared response. A spectral factorization step is then used to derive a minimum-phase rational transfer function that approximates the original magnitude response. Specifically, the magnitude-squared response is fitted using a pole-residue form. Ideally, the resulting poles and zeros form complex conjugate pairs that are symmetrically distributed across the left and right halves of the complex plane. The final model is obtained by retaining only those poles and zeros in the left half of the complex plane, ensuring a stable and causal minimum-phase system. However, in practice, challenges arise during the modeling iterations. In particular, some of the computed zeros may lie directly on the imaginary axis, making it ambiguous or even impossible to determine their left- or right-half-plane affiliation. This problem stems from a mismatch in constraints: while the magnitude-squared response must be real and non-negative along the frequency axis, the rational function used for fitting is not inherently guaranteed to satisfy these properties [8,9]. As a result, the spectral factorization step may produce ill-posed or physically inconsistent results. Furthermore, PICs are typically modeled using baseband equivalents [10–12], whose frequency responses generally do not satisfy Hermitian symmetry (e.g., even magnitude and odd phase). Consequently, the MVF method, originally developed for systems exhibiting Hermitian symmetry, is not directly applicable to the modeling of photonic circuits.

This paper proposes a robust and flexible modeling method for constructing rational models of PICs on their magnitude data. The proposed approach begins by fitting the magnitude-squared frequency response and then introduces a dedicated procedure to properly handle imaginary zeros. This enables the synthesis of a consistent phase response corresponding to the given magnitude data. By combining the synthesized phase with the magnitude, any frequency response approximation technique, such as the complex vector fitting (CVF) method [10], can then be applied to construct the final rational model. The modeling process consists of two independent steps: phase synthesis and frequency response fitting. In the first step, a high-order transfer function with many poles can be potentially used to accurately approximate the magnitude-squared data. In the second step, the number of poles required for full response approximation using CVF is independent of the pole count used in the first step. This decoupling enables flexible

and independent control of model complexity and accuracy, thereby improving modeling efficiency and enhancing adaptability in photonic applications.

This paper is organized as follows. Section 2 introduces the MVF algorithm and reviews the challenges encountered during the modeling process. Section 3 details the proposed approach to baseband equivalent modeling of PICs from magnitude-only responses. To demonstrate the effectiveness of the proposed method, Section 4 presents three case studies involving magnitude-only data from PICs. Finally, Section 5 discusses the limitations and time-domain performances of the proposed models, while Section 6 concludes the paper. To avoid confusion, the term “physically consistent” in this paper refers to a phase that is consistent with the Kramers–Kronig relations for a given magnitude response, rather than matching the true physical phase of PICs.

2. BACKGROUND AND PROBLEM REVIEW

A. Review of the MVF Modeling Technique

The MVF algorithm was first introduced and investigated in Ref. [8]. The method begins by constructing a rational approximation of the magnitude-squared frequency response. It then applies spectral factorization to the resulting rational function to derive a minimum-phase transfer function that approximates the original magnitude response. This section provides a brief overview of the MVF technique.

The method assumes the existence of a pole-zero transfer function whose magnitude response matches that of the target system. Specifically, it is assumed that

$$H(s) = \sqrt{d} \frac{\prod_{n=1}^N (s - z_n)(s - z_n^*)}{\prod_{n=1}^N (s - p_n)(s - p_n^*)}, \quad (1)$$

where p_n and z_n are the poles and zeros, respectively, s is the Laplace variable, and $\sqrt{d}$ is the gain. The superscript symbol $*$ represents the complex conjugate operation. Note that all poles and zeros are either real or occur in complex conjugate pairs, and they are located in the left half of the complex plane. For clarity, only the complex conjugate pairs are shown in Eq. (1).

Then, the magnitude-squared response of $H(s)$ [i.e., the intensity $I(s)$] is given by

$$\begin{aligned} I(s) &= |H(s)|^2 = H(s) \cdot H^*(s) \\ &= d \frac{\prod_{n=1}^N (s - z_n)(s - z_n^*)(s + z_n)(s + z_n^*)}{\prod_{n=1}^N (s - p_n)(s - p_n^*)(s + p_n)(s + p_n^*)}, \end{aligned} \quad (2)$$

when it is restricted to the imaginary axis $s = j\omega$. Equation (2) indicates that the transfer function of $I(s)$ includes complex conjugate pairs of poles and zeros located in the left half of the complex plane, along with their symmetric counterparts in the right half. The following key idea is to apply spectral factorization to find the minimum-phase rational model for $|H(s)|$ by discarding the right-hand-side poles/zeros.

Now, the key step is to determine the transfer function [Eq. (2)], which can be represented in a pole-residue form as follows:

$$\begin{aligned}
 I(s) &= \sum_{n=1}^N \left(\frac{c_n}{s-p_n} + \frac{-c_n}{s+p_n} + \frac{c_n^*}{s-p_n^*} + \frac{-c_n^*}{s+p_n^*} \right) + d \\
 &= \sum_{n=1}^N \left(\frac{2c_n p_n}{s^2 - p_n^2} + \frac{2c_n^* p_n^*}{s^2 - (p_n^*)^2} \right) + d.
 \end{aligned} \quad (3)$$

Note that the representation in Eq. (2) is non-negative by construction. When it is converted into the form of Eq. (3), the resulting representation also remains non-negative. However, through simple mathematical manipulations, it can be shown that the pole-residue form itself does not inherently guarantee non-negativity. In this context, Eq. (2) serves as a sufficient but not necessary condition for the positivity of Eq. (3). This issue gives rise to critical problems, as detailed in the next section.

Thanks to the vector fitting (VF) algorithm [13], this problem can be converted to solving the following linear least square (LS) problem:

$$(I\sigma)_{\text{fit}}(s) - I(s)\sigma(s) \approx 0, \quad (4)$$

where $(I\sigma)_{\text{fit}}(s)$ and $\sigma(s)$ are

$$(I\sigma)_{\text{fit}}(s) = \sum_{n=1}^N \left(\frac{2c_n p_n^{(0)}}{s^2 - (p_n^{(0)})^2} + \frac{2c_n^* (p_n^{(0)})^*}{s^2 - (p_n^{(0)*})^2} \right) + d, \quad (5)$$

$$\sigma(s) = \sum_{n=1}^N \left(\frac{2\tilde{c}_n p_n^{(0)}}{s^2 - (p_n^{(0)})^2} + \frac{2\tilde{c}_n^* (p_n^{(0)})^*}{s^2 - (p_n^{(0)*})^2} \right) + \tilde{d}. \quad (6)$$

Note that Eq. (4) is enforced at each frequency sample of $|H(s)|^2$. This results in a system of equations with unknowns $\{c_n\}$, d , $\{\tilde{c}_n\}$, and $\tilde{d}$, while $\{p_n^{(0)}\}$ denotes the initial guess for the set of poles. After solving this LS problem, the zeros of $\sigma(s)$ are extracted and used as the updated set of poles in the next iteration of Eq. (4). This iterative process continues until a pre-defined convergence criterion is met. Once converged, the final pole-residue model [Eq. (3)] for $I(s)$ can be obtained.

B. Problem Review

By definition, the magnitude-squared response $I(s)$ that is expressed in the pole-zero form in Eq. (2) is non-negative. The MVF method, however, does not construct the model directly in the form of Eq. (2). Instead, it identifies the poles and residues of the pole-residue representation in Eq. (3) through an iterative procedure, which is subsequently expected to be converted back into the pole-zero form in Eq. (2). A key issue then arises: unlike the pole-zero representation, the pole-residue form in Eq. (3) does not inherently guarantee non-negativity. The same limitation also applies to the function $\sigma(s)$. This leads to two problems.

(i) In each iteration, it is necessary to calculate the zeros of $\sigma(s)$ as the poles for the next iteration from the residues $\{\tilde{c}_n\}$. However, it can be verified that $\sigma(s)$ in Eq. (6) can only be converted into the following form:

$$\sigma(s) = \frac{\prod_{n=1}^{2N} (s^2 - \tilde{z}_n^2)}{\prod_{n=1}^{2N} (s^2 - p_n^2)}, \quad (7)$$

where $\tilde{z}_n$ is the calculated zeros, rather than in the following form:

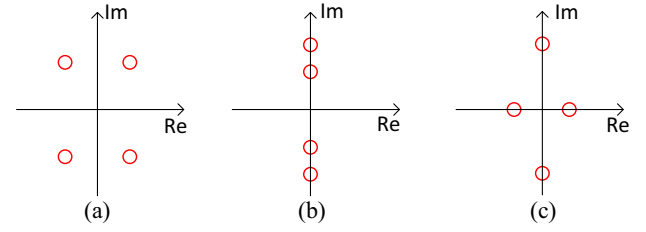


Fig. 2. (a) The desired zero distribution in quadruples; (b) and (c) the undesired zero distributions in Eqs. (6) and (7).

$$\sigma(s) = \frac{\prod_{n=1}^N (s^2 - \tilde{z}_n^2)(s^2 - (\tilde{z}_n^*)^2)}{\prod_{n=1}^N (s^2 - p_n^2)(s^2 - (p_n^*)^2)}. \quad (8)$$

A crucial difference between Eqs. (7) and (8) is that the latter is guaranteed to be non-negative, while the former is not. As a result, although we expect the zeros to be distributed as shown in Fig. 2(a), they may instead appear as in Figs. 2(b) and 2(c). In such cases, purely imaginary zeros cannot be adopted as the new poles in the subsequent iteration. To address this, the work in Ref. [8] proposes mapping imaginary zeros to real values. Specifically, if the zeros are located at $\pm j\alpha$, they are forcibly relocated to $\pm\alpha$. While this mapping may introduce a relatively large perturbation, the iterative process often compensates by adjusting the positions of other poles [8]. Although this technique has proven effective in modeling RF/microwave circuits, we will show in Section 3.B that it is not applicable to baseband modeling of PICs.

(ii) After a certain number of iterations, Eq. (4) typically converges to the desired level of accuracy. At this point, the poles and residues of $I(s)$ can be determined, and the next step is to compute the zeros and perform spectral factorization. This involves retaining only the poles and zeros located in the left half of the complex plane and discarding those in the right half. However, a previously mentioned issue may arise again: some zeros may lie exactly on the imaginary axis. In such cases, relocating these zeros to the real axis is not a viable option, as it may introduce significant approximation errors. An alternative approach, as proposed in Ref. [9], is to incorporate the following additional constraints into the LS problem [Eq. (3)] when calculating residues with known poles:

$$\begin{cases} I(s_k) = |H(s_k)|^2 \geq 0, & k = 1, 2, \dots, K, \\ d \geq 0 \end{cases}, \quad (9)$$

where K represents the number of frequency points in s .

This leads to a constrained ℓ_2 -norm minimization problem, which must be solved using convex constrained optimization techniques [9]. However, this approach only enforces the constraints at a finite set of discrete frequency points within a limited bandwidth. As a result, it is still possible for $I(s) < 0$ to occur between these frequency points or outside the specified modeling range. Furthermore, in practice, the introduction of such constraints may reduce the accuracy of the resulting approximation. To address this issue, the following section presents a robust and effective alternative solution.

3. PROPOSED MODELING METHOD FOR PICs BASED ON MAGNITUDE-ONLY RESPONSES

A. Baseband Representations of PICs and the CVF Modeling Technique

In applications like optical interconnects and photonic neural networks, PICs typically process optical signals that carry information through magnitude and/or phase modulation. Such signals can be expressed in the following general form:

$$u(t) = A(t) \cos(2\pi f_c t + \phi(t)), \quad (10)$$

where $A(t)$ and $\phi(t)$ denote the time-varying amplitude and phase modulation, respectively, while f_c represents the optical carrier frequency. For example, the carrier frequency f_c can be as high as 193.4 THz, which lies in the optical C-band and corresponds to a wavelength of approximately 1.55 μm . Direct time-domain simulation of such high-frequency signals and their associated photonic systems is computationally impractical, as it would require extremely small time steps. Therefore, the complex baseband equivalent of the optical signal [Eq. (10)] is typically used for analysis [14–16],

$$u_b(t) = A(t)e^{j\phi(t)}, \quad (11)$$

where the optical carrier is removed.

Meanwhile, PICs also need to be modeled in the baseband rather than at their original optical carrier frequencies. The baseband equivalent system can be obtained by downshifting the frequency response over the positive frequency range, as illustrated in Fig. 3. It is important to note that passive PICs are real-valued, linear, and time-invariant physical systems, and their frequency responses inherently satisfy Hermitian symmetry. In contrast, the baseband equivalent systems are purely mathematical representations and do not obey this symmetry constraint. Due to this fundamental difference, the VF algorithm originally developed to model real-valued, linear time-invariant (LTI) physical systems [13] is not applicable to baseband equivalent systems. To address this gap, the CVF method was introduced [10]. The key distinction between CVF and VF lies in the handling of poles and residues: unlike VF, CVF does not require them to appear in complex conjugate pairs. Given the magnitude and phase responses of a PIC, the CVF method can construct a baseband equivalent model expressed as

$$P(s) = \sum_{n=1}^N \frac{c_n}{s - p_n} + d, \quad (12)$$

where p_n and c_n are either real or complex. In Eq. (12), $P(s)$ denotes the baseband model and is used to distinguish it from the original transfer function $H(s)$, whose poles are real or

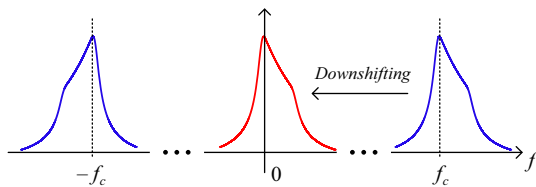


Fig. 3. The baseband equivalent system for PICs. The blue curves represent the symmetric magnitude frequency response of the PICs, while the red curve shows the asymmetric magnitude response of the corresponding baseband equivalent system.

occur in complex conjugate pairs. The CVF, thus, provides a baseband macromodeling framework for linear passive PICs. However, the method fails when phase information is not available. Note that, the rational model [Eq. (12)] can be readily converted to a time-domain state-space model for circuit-level time-domain analysis [10].

B. Modeling PICs from Magnitude-Only Responses

Considering that baseband frequency responses for PICs are not Hermitian symmetric, the existing MVF method has to be modified to adapt to the baseband scenario. First, we assume PICs can be represented with the baseband equivalent model,

$$P(s) = \sqrt{d} \frac{\prod_{n=1}^N (s - z_n)}{\prod_{n=1}^N (s - p_n)}, \quad (13)$$

where p_n and z_n are the poles and zeros, respectively, and $\sqrt{d}$ is the gain. Note that p_n and z_n are real or complex and do not necessarily have to be in complex conjugate pairs. Then, the magnitude-squared response of $P(s)$ is given by

$$I(s) = |P(s)|^2 = P(s) \cdot P^*(s) = d \frac{\prod_{n=1}^N (s - z_n)(s + z_n^*)}{\prod_{n=1}^N (s - p_n)(s + p_n^*)}, \quad (14)$$

which corresponds to the optical power (or intensity) response of the device.

It is important to observe the difference between Eq. (2) for RF/microwave circuits and Eq. (14) for PICs. The poles and zeros in Eq. (2) are in quadruples, as shown in Fig. 2(a), while those in Eq. (14) are only symmetric with respect to the imaginary axis.

In order to determine the transfer function [Eq. (14)], it is represented in a pole-residue form,

$$I(s) = \sum_{n=1}^N \left(\frac{c_n}{s - p_n} + \frac{-c_n^*}{s + p_n^*} \right) + d. \quad (15)$$

Then, a new LS problem can be formulated,

$$(I\rho)_{\text{fit}}(s) - I(s)\rho(s) \approx 0, \quad (16)$$

where $(I\rho)_{\text{fit}}(s)$ and $\rho(s)$ are

$$(I\rho)_{\text{fit}}(s) = \sum_{n=1}^N \left(\frac{c_n}{s - p_n^{(0)}} + \frac{-c_n^*}{s + (p_n^{(0)})^*} \right) + d, \quad (17)$$

$$\rho(s) = \sum_{n=1}^N \left(\frac{\tilde{c}_n}{s - p_n^{(0)}} + \frac{-\tilde{c}_n^*}{s + (p_n^{(0)})^*} \right) + \tilde{d}. \quad (18)$$

Following a similar procedure to MVF, solving the LS problem gives $\{\tilde{c}_n\}$ and $\tilde{d}$, from which the zeros need to be computed and adopted as the new poles for the subsequent iteration.

To determine the zeros, a similar approach to that used in VF is adopted. The zeros are computed as the eigenvalues of $\mathbf{A} - \mathbf{B}\mathbf{C}\tilde{\mathbf{d}}^{-1}$, where $\mathbf{A}$ is a diagonal matrix containing the poles $p_n^{(i)}$, $\mathbf{B}$ is a column vector of ones, and $\mathbf{C}$ is a row vector containing the residues $\tilde{c}_n^{(i)}$. It is important to note that directly computing the eigenvalues may yield results that are not symmetric with respect to the imaginary axis due to numerical errors. To avoid this, it is recommended to convert

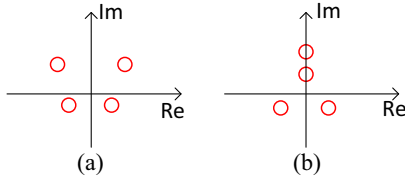


Fig. 4. Possible zero distributions when modeling PICs. (a) Symmetric zeros; (b) imaginary zeros.

$A - BC\tilde{d}^{-1}$ into a real matrix prior to performing eigenvalue decomposition.

After computing the zeros, it is observed that most zeros are symmetric about the imaginary axis, while some lie directly on the imaginary axis, as illustrated in Fig. 4. Meanwhile, $\rho(s)$ can be expressed as

$$\rho(s) = \frac{\prod_{n=1}^{N-L} (s - z_n)(s + z_n^*)}{\prod_{n=1}^N (s - p_n)(s + p_n^*)} \cdot \zeta(s), \quad (19)$$

where

$$\zeta(s) = \prod_{l=1}^L (s - jR_{2l-1})(s - jR_{2l}), \quad (20)$$

where jR_{2l-1} and jR_{2l} are the adjacent imaginary zeros.

In MVF, the imaginary zeros are mapped to real numbers by omitting the imaginary unit j , which later are adopted as the poles for the next iteration. However, this approach is not applicable to baseband systems since R_{2l-1} is typically not equal to R_{2l} , whereas poles must be symmetric about the imaginary axis. Therefore, a different treatment for these zeros is introduced,

$$jR_{2l-1}, jR_{2l} \Rightarrow \alpha + j\beta, -\alpha + j\beta, \quad (21)$$

where $\alpha = 0.01\beta$ and $\beta = (R_{2l-1} + R_{2l})/2$. This adjustment may degrade the accuracy of the LS problem; however, the resulting error can be compensated in the next iteration. Notably, the method of modifying the zeros to ensure symmetry with respect to the imaginary axis is not unique. For example, it can alternatively be implemented as follows:

$$jR_{2l-1}, jR_{2l} \Rightarrow \beta, -\beta. \quad (22)$$

We evaluated the two solutions in terms of the magnitude response of a Mach–Zehnder interferometer (MZI). In the test, $I(s)$ was approximated using 10 iterations of pole relocation, during which the two options were applied. The accuracy was then compared as a function of the number of poles, as shown in Fig. 5. It is evident that Eq. (21) demonstrates a faster convergence. It is important to remark that we select $\alpha = 0.01\beta$ based on tests concerning convergence rate and accuracy. Thus, $\alpha = 0.01\beta$ is recommended as a heuristic choice rather than a fixed rule.

C. Phase Synthesis for Constructing Rational Models of PICs

When a predefined number of iterations is reached or the desired accuracy is achieved, the process terminates, yielding a transfer function in pole-residue form as given in Eq. (15). And $I(s)$ is subsequently expressed in the pole-zero form, assuming the presence of imaginary zeros,

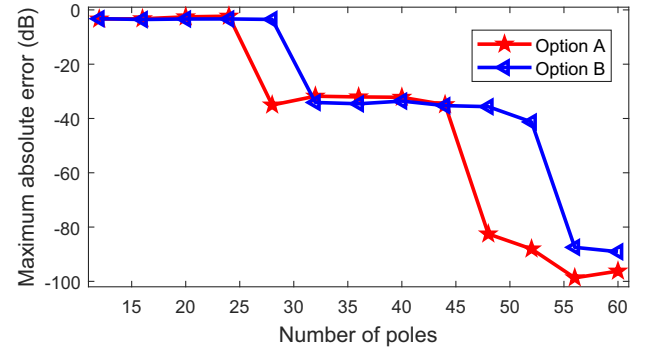


Fig. 5. Convergence rate of approximating $|P(s)|^2$ using Option A [Eq. (21)] and Option B [Eq. (22)].

$$I(s) = \frac{\prod_{n=1}^{N-L} (s - z_n)(s + z_n^*)}{\prod_{n=1}^N (s - p_n)(s + p_n^*)} \cdot \zeta(s), \quad (23)$$

where

$$\zeta(s) = d \prod_{l=1}^L (s - jR_{2l-1})(s - jR_{2l}). \quad (24)$$

jR_{2l-1} and jR_{2l} represent the imaginary zeros in $I(s)$.

When there are imaginary zeros in Eq. (23), their total number can be $2L$, $L = 0, 1, \dots$. Any imaginary zero defines a cross-over frequency where $I(s)$ changes from being positive to negative, or vice versa. Correspondingly, the function $I(s)$ becomes negative between pairs of these zeros, as shown in Fig. 6(a), which depicts four zeros, two located within the modeling frequency range $[-\omega_{\max}, \omega_{\max}]$ and two outside it. It is important to note that the negative portions of $I(s)$ must be very small between the zeros inside the modeling range since $I(s)$ closely approximates the non-negative magnitude-squared data. This behavior is exemplified in the interval between R_1 and R_2 . On the other hand, when the zeros lie outside the modeling range, the corresponding negative values of $I(s)$ can vary in magnitude, and they may be small or relatively large, as illustrated in the interval between R_3 and R_4 . In the case of $d < 0$ in Eq. (24), the response of $I(s)$ can be illustrated by Fig. 6(b). In both cases, when R_{2l-1} and R_{2l} lie within the modeling frequency range, the negative portion of $I(s)$ is relatively small.

Note that, the zero adjustment in Eqs. (21) and (22) can only be applied to $\rho(s)$ during the iteration process. Applying them directly to Eq. (23) results in large approximation errors in $I(s)$.

For both cases, the imaginary zeros outside the modeling range are denoted by $j\hat{R}_l$ and are discarded, while the imaginary zeros within the modeling range are denoted by $j\tilde{R}_l$ and then are subsequently modified,

$$j\tilde{R}_{2l-1}, j\tilde{R}_{2l} \Rightarrow \varepsilon + j\frac{\tilde{R}_{2l-1} + \tilde{R}_{2l}}{2}, -\varepsilon + j\frac{\tilde{R}_{2l-1} + \tilde{R}_{2l}}{2}, \quad (25)$$

where $j\tilde{R}_{2l-1}$ and $j\tilde{R}_{2l}$ are two adjacent imaginary zeros in the modeling range, $\varepsilon \rightarrow 0^+$, which can be 10^{-10} in practice.

Once the imaginary zeros have been properly addressed, the transfer function [Eq. (23)] becomes

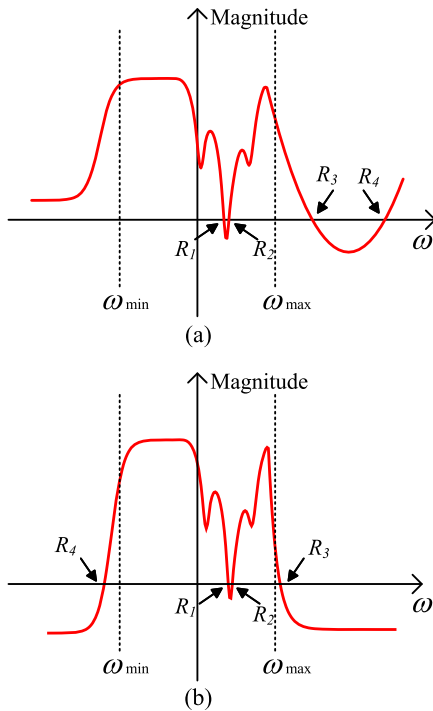


Fig. 6. Response of $I(s)$ with imaginary zeros. (a) $I(s)$ becomes negative between pairs of the imaginary zeros; (b) distribution of imaginary zeros when $d < 0$.

$$\tilde{I}(s) = (-1)^{\tilde{L}} \frac{\prod_{n=1}^{N-\tilde{L}} (s - z_n)(s + z_n^*)}{\prod_{n=1}^N (s - p_n)(s + p_n^*)} \cdot \tilde{\zeta}(s), \quad (26)$$

where

$$\tilde{\zeta}(s) = \prod_{l=1}^{\tilde{L}} (s - Z_l)(s + Z_l^*), \quad (27)$$

where $Z_l = \varepsilon + j \frac{\tilde{R}_{2l-1} + \tilde{R}_{2l}}{2}$. The values $2\tilde{L}$ and $2\hat{L}$ represent the total number of imaginary zeros located within and outside the modeling frequency range, respectively, such that $L = \tilde{L} + \hat{L}$. All poles and zeros of $\tilde{I}(s)$ are symmetrically distributed with respect to the imaginary axis. The term $(-1)^{\tilde{L}}$ in Eq. (26) is introduced to ensure that $\tilde{I}(s)$ remains non-negative and can be factorized as $\tilde{P}(s)\tilde{P}^*(s)$ for $s = j\omega$.

Now, the relation between $\tilde{I}(s)$ and $I(s)$ can be described by

$$I(s) \approx |\tilde{I}(s)| \cdot (-1)^{\hat{L}} d \prod_{l=1}^{2\hat{L}} (s - j\hat{R}_l). \quad (28)$$

When considering $s = j\omega$, the relation can be updated as

$$I(j\omega) \approx \tilde{I}(j\omega) \cdot d \prod_{l=1}^{2\hat{L}} (\omega - \hat{R}_l). \quad (29)$$

It is straightforward to verify that $d \prod_{l=1}^{2\hat{L}} (\omega - \hat{R}_l)$ remains consistently positive within the modeling range $[\omega_{\min}, \omega_{\max}]$. Based on this, the following relation can be established:

$$P(j\omega) \approx \tilde{P}(j\omega) \sqrt{d \prod_{l=1}^{2\hat{L}} (\omega - \hat{R}_l)}, \quad \omega \in [\omega_{\min}, \omega_{\max}], \quad (30)$$

where $\tilde{P}(j\omega)\tilde{P}^*(j\omega) = \tilde{I}(j\omega)$.

The relation in Eq. (30) indicates that, although $P(j\omega)$ and $\tilde{P}(j\omega)$ have different magnitudes, they share the same phase response within the modeling range since $d \prod_{l=1}^{2\hat{L}} (\omega - \hat{R}_l) > 0$. As a result, the phase of $\tilde{P}(j\omega)$ can be used as an approximation of the phase of $P(j\omega)$.

Finally, spectral factorization for $\tilde{I}(s)$ can be immediately conducted to obtain a minimum-phase transfer function $\tilde{P}(s)$ by removing the poles and zeros in the right-half complex plane. Then, the phase response of $\tilde{P}(s)$ can be effectively paired with the magnitude response $|P(s)|$ to construct the full frequency response. Note that, based on Eq. (30), it is straightforward to verify that the synthesized phase is consistent with the Kramers–Kronig relations corresponding to the given magnitude data [17]. Consequently, the complete frequency response $P(s)$ can be constructed as

$$P(s_k) = |P(s_k)| e^{j \arg(\tilde{P}(s_k))}, \quad k = 1, 2, \dots, K. \quad (31)$$

Subsequently, CVF can be applied to construct a rational model in the form of Eq. (12) for the constructed tabulated data $P(s_k)$.

4. EXAMPLES

In this section, three representative application examples are presented to evaluate the effectiveness and versatility of the proposed modeling method in PICs. The first two examples are based on simulated circuits, from which we have both the magnitude and phase response, allowing a proper comparison with a reference model. The third example is based on measured data. All modeling procedures are performed using MATLAB on a standard personal computer equipped with an Intel Core i7 processor and 16 GB of RAM.

A. Arrayed Waveguide Grating

Arrayed waveguide gratings (AWGs) are widely used as optical (de)multiplexers in wavelength-division multiplexed (WDM) systems. In this example, the proposed technique is applied to the AWG presented in Ref. [18] to demonstrate its applicability and effectiveness in modeling PICs. The structure and design parameters of the AWG are provided in Ref. [18]. The transmission response of an arbitrary channel over the frequency range $f \in [193.17 \text{ THz}, 193.91 \text{ THz}]$ is selected as the target magnitude response $|P(s)|$. According to the baseband modeling approach, this magnitude response is shifted to baseband with respect to a carrier frequency of $f_c = 193.54 \text{ THz}$. The proposed method is then applied to the resulting baseband magnitude data over the baseband frequency range $f \in [-370 \text{ GHz}, 370 \text{ GHz}]$. Figure 7 shows the approximation of $I(s)$, which achieves a maximum absolute error of less than -125 dB when 30 poles are used. It is worth noting that the passband center of the selected transmission is not aligned with $f_c = 193.54 \text{ THz}$. This configuration is intentional, as it allows for the analysis of inter-channel crosstalk from the selected channel to the one centered at

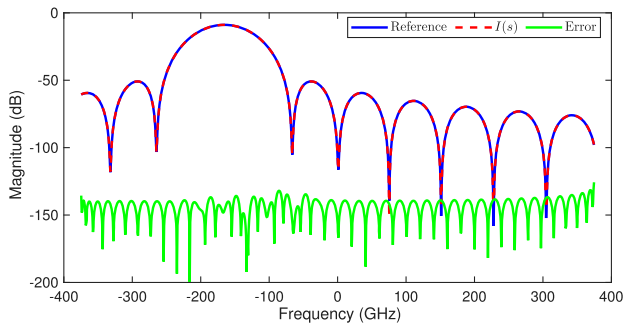


Fig. 7. Example A. The approximation results of $I(s)$.

$f_c = 193.54$ THz. For additional details about the AWG, interested readers can refer to Ref. [18].

Then, the transfer function of $I(s)$ in pole-residue form is converted into its pole-zero representation. The distribution of zeros is shown in Fig. 8, where six imaginary zeros— $j9.482 \times 10^{11}$, $j9.525 \times 10^{11}$, $j1.428 \times 10^{12}$, $j1.434 \times 10^{12}$, $j1.912 \times 10^{12}$, $j1.919 \times 10^{12}$ —are within the modeling range.

To better observe the crossover frequency points, $I(s)$ is plotted in Fig. 9 using a linear scale instead of dB and over an extended frequency range. It can be observed that the six imaginary zeros appear in pairs, and the portions of $I(s)$ between each pair are negative.

Then, the six imaginary zeros are adjusted according to Eq. (25). After these operations, the modified transfer function $\tilde{I}(s)$ is obtained. Spectral factorization is then applied to derive $\tilde{P}(s)$. Finally, the phase response of $P(s)$ is extracted from $\tilde{P}(s)$ and compared with the original phase of the transmission, as shown in Fig. 10. It is evident that the synthesized phase differs significantly from the original phase. From a theoretical standpoint, it is impossible to uniquely recover the phase using magnitude-only data. In Section 5, we will discuss this issue in detail and analyze how the differences in phase affect the time-domain simulation results.

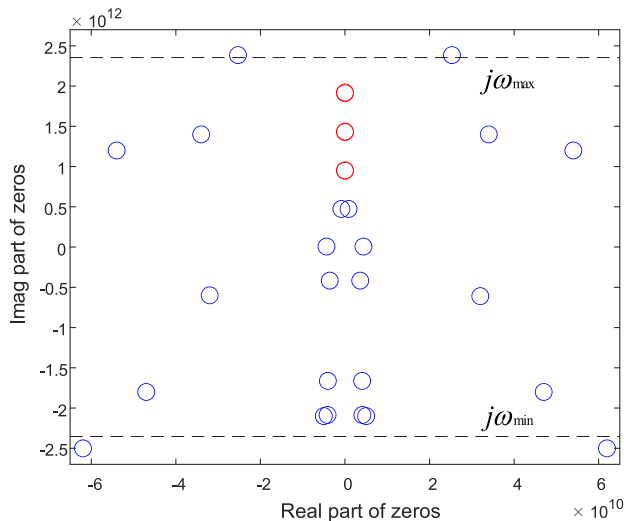


Fig. 8. Example A. The distribution of zeros of $I(s)$. The blue circles represent the zeros that are symmetric about the complex axis while the red circles indicate the imaginary zeros.

The full frequency response is then reconstructed by pairing the synthesized phase with the magnitude data. Subsequently, the CVF algorithm is applied to the full frequency response to construct a rational model using 14 poles. The resulting model achieves a maximum absolute error of -77 dB, as shown in Fig. 11. It should be noted that Fig. 11 presents the error between the CVF model and the constructed full frequency response (original magnitude with synthesized phase), not the original full frequency response. This result verifies that the synthesized phase is consistent with the Kramers–Kronig relation associated with the given magnitude data, as evidenced by the very small modeling error. This example demonstrates that the proposed method is capable of constructing a rational model whose magnitude response is almost identical to that of the PIC under study.

Furthermore, to explore the relationship between modeling complexity and accuracy for $I(s)$ and $|P(s)|$, we conducted a detailed investigation. First, $I(s)$ was fitted using different numbers of poles, each resulting in varying levels of accuracy and corresponding synthesized phase responses. These synthesized phases were then paired with the original $|P(s)|$ data, and CVF was applied with different pole counts. The results are illustrated in Fig. 12. When $I(s)$ is poorly approximated, such as with only 10 or 18 poles, it becomes challenging to construct an accurate model for $|P(s)|$ using CVF, even with a large number of poles. In contrast, when $I(s)$ is accurately fitted with 30 or more poles, CVF is able to generate a highly accurate model for $|P(s)|$ using fewer than 14 poles. This clearly demonstrates that the final rational model of $P(s)$ can be more compact than models obtained by directly factorizing the intermediate representation of $I(s)$.

This example confirms the validity of the analysis concerning imaginary zeros and the associated negative regions in $I(s)$. It further shows that, given the magnitude response of a PIC, the proposed method can construct a model with a magnitude response that closely matches that of the PIC under study.

B. Five-Ring Resonator

This section investigates an SOI-based five-ring resonator reported in Ref. [10]. The structure consists of five coupled microring resonators and is shown in Fig. 13. The frequency response data are obtained via finite-difference time-domain simulations. The transmission S_{12} in the S -matrix is arbitrarily selected over the frequency range $f \in [187.37$ THz, 199.86 THz], corresponding to a wavelength range of $\lambda \in [1.5$ μm , 1.6 $\mu\text{m}]$. Only the magnitude response, $|S_{12}|$, is used as the target response $|P(s)|$. The proposed method is then applied to model $|S_{12}|$ within the baseband frequency range $f \in [-6.245$ THz, 6.245 THz].

Figure 14 presents the approximation of $I(s)$, achieving a maximum absolute error below -43 dB with 100 poles. As will be demonstrated later, even when $I(s)$ is approximated with lower accuracy, it is still possible to construct an accurate model for $|P(s)|$.

Then, the transfer function of $I(s)$ in pole-residue form is converted into its pole-zero representation. The distribution of zeros is shown in Fig. 15, where two imaginary zeros $j5.33 \times 10^{13}$, $-j4.15 \times 10^{13}$ appear outside the modeling range.

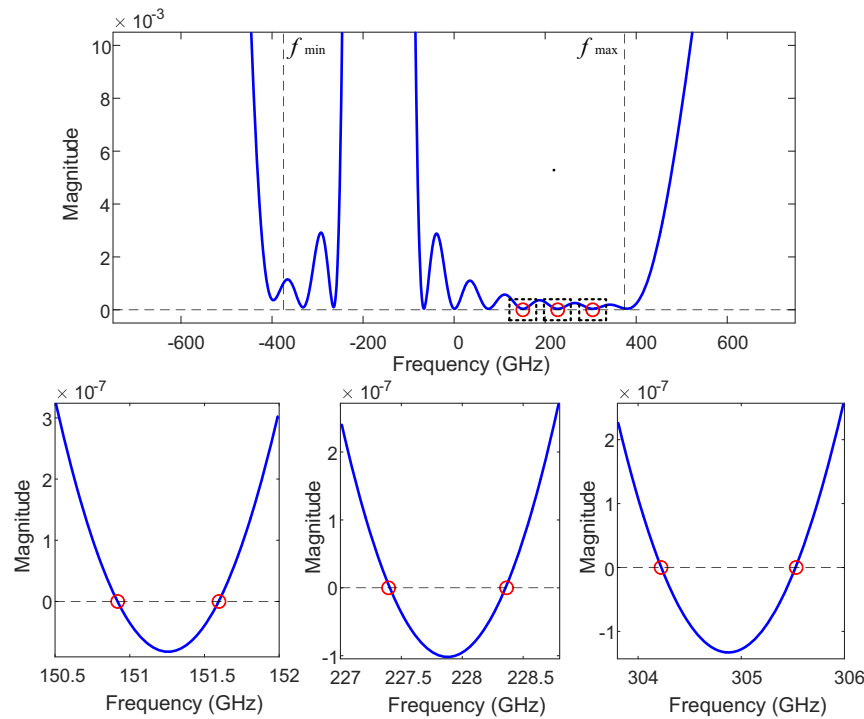


Fig. 9. Example A. Response of $I(s)$ with six imaginary zeros. The bottom panels show enlarged views of the regions highlighted in the top panel.

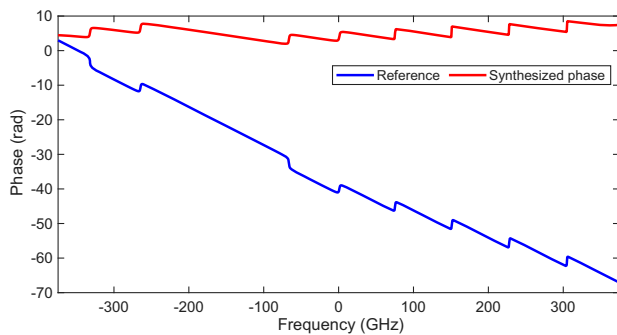


Fig. 10. Example A. The comparison of the original phase and the synthesized phase by the proposed method.

To better illustrate the relation of imaginary zeros and the negativeness of $I(s)$, $I(s)$ over a wider frequency range is plotted in Fig. 16 using a linear scale.

Then, the two zeros are discarded to derive the transfer function $\tilde{I}(s)$. Spectral factorization is then applied to derive $\tilde{P}(s)$. Finally, the phase response of $P(s)$ is extracted from $\tilde{P}(s)$ and compared with the original phase of S_{12} , as shown in Fig. 17.

The full frequency response is then reconstructed by pairing the synthesized phase with the magnitude data. Subsequently, the CVF algorithm is applied to the complete frequency response to construct a rational model using 40 poles. The resulting model achieves a maximum absolute error of -51 dB, which is notably more accurate than the approximation of $I(s)$, as shown in Fig. 18. This example further demonstrates the

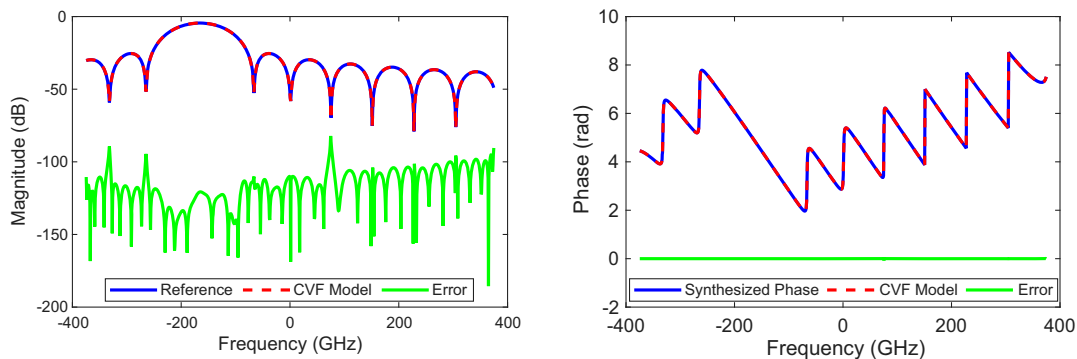


Fig. 11. Example A. The CVF modeling accuracy of the constructed full frequency response (original magnitude and synthesized phase).

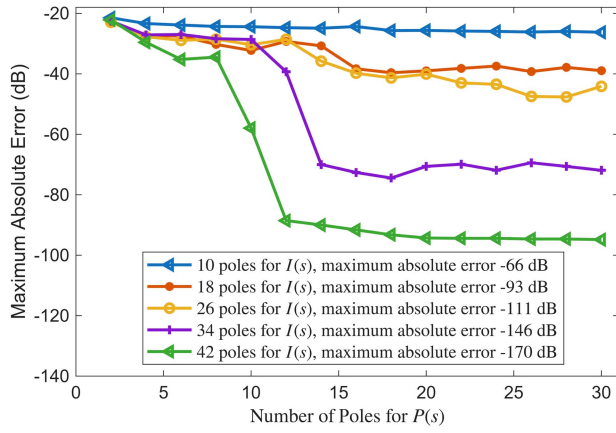


Fig. 12. Example A. Comparison of modeling accuracy and complexity for $|P(s)|$ and $I(s)$. The magnitude-squared response $I(s)$ is first fitted using 10, 18, 26, 34, and 42 poles. Based on the corresponding synthesized phase responses, $|P(s)|$ is then modeled using CVF with an increasing number of poles.



Fig. 13. Example B. The structure of the five-ring resonator filter.

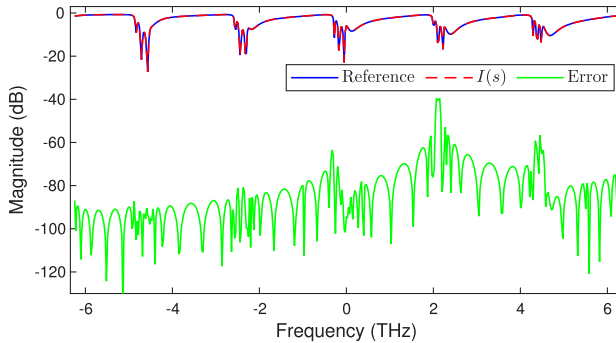


Fig. 14. Example B. The approximation results of $I(s)$.

effectiveness of the proposed method for modeling the magnitude responses of PICs and confirms that the final rational model of $|P(s)|$ can achieve higher accuracy with fewer poles, less than half the number used in the approximation of $I(s)$.

C. Mach-Zehnder Interferometer Loaded with Four Rings

In this section, the proposed method is applied to an MZI structure incorporating two cascaded ring resonators in each arm, as illustrated in Fig. 19 and presented in Ref. [19]. The cascaded rings in each arm act as a second-order all-pass

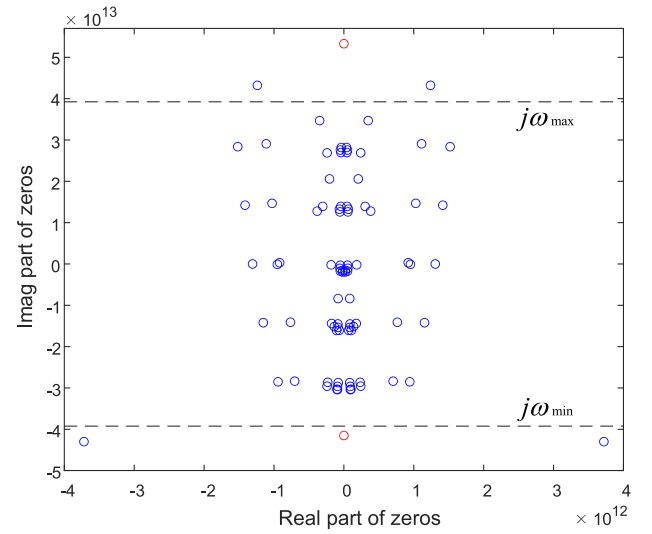


Fig. 15. Example B. The distribution of zeros of $I(s)$. The blue circles represent the zeros that are symmetric about the complex axis while the red circles indicate the imaginary zeros.

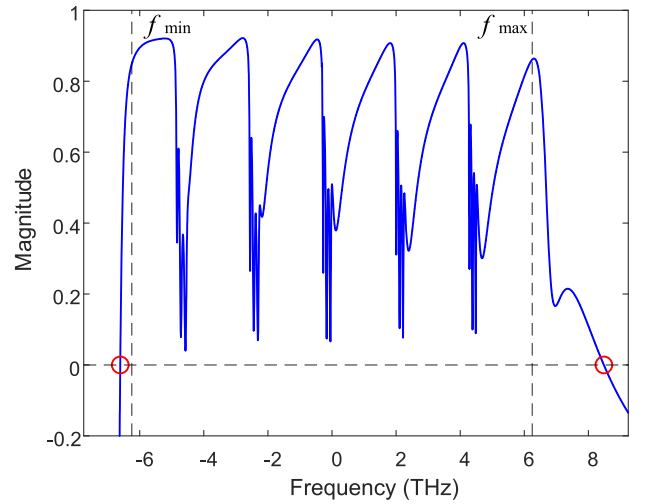


Fig. 16. Example B. Response of $I(s)$ with two imaginary zeros.

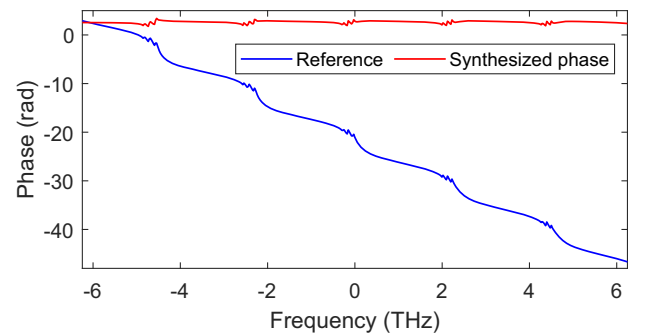


Fig. 17. Example B. The comparison of the original phase and the synthesized phase by the proposed method.

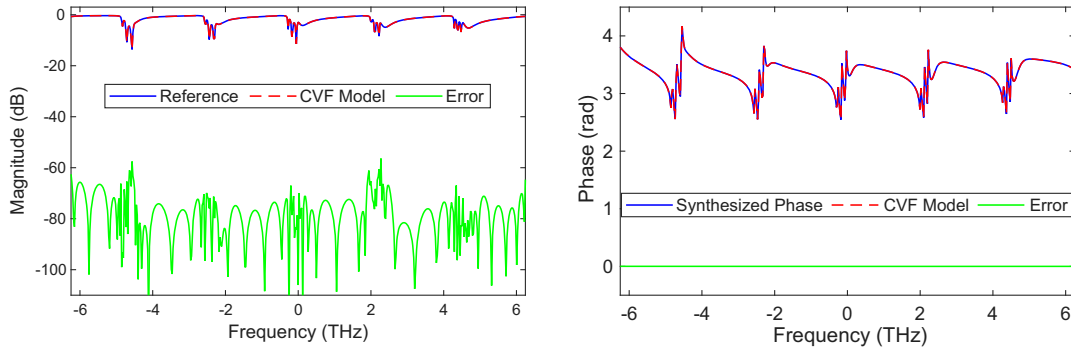


Fig. 18. Example B. The CVF modeling accuracy of the constructed full frequency response (original magnitude and synthesized phase).

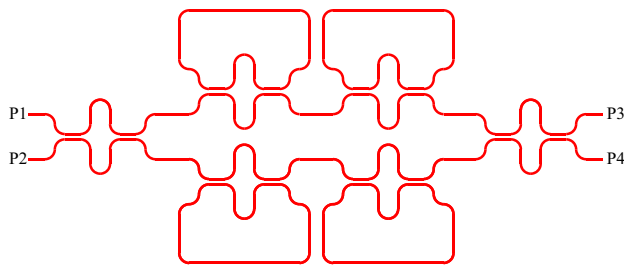


Fig. 19. Example C. Schematic of the four rings loaded MZI.

filter in the lossless case, while the combiners mix the signals from the two arms. The circuit was designed using Luceda IPKISS and fabricated on the imec iSiPP50G silicon photonic platform through multi-project wafer services [19]. The magnitude response of the device was measured in the laboratory and is available at Ref. [20], whereas the phase response is unknown.

It is worth noting that the measured magnitude response often contains noise-induced perturbations. Therefore, preprocessing is required before model construction. In this work, a Savitzky–Golay filter [21] is employed due to its superior smoothing capability. This filtering procedure effectively suppresses measurement noise while preserving important high-frequency features of the response. Note that the filter parameters (e.g., window length, polynomial order) affect the trade-off between noise suppression and feature preservation, and they should be chosen based on the intended application. An arbitrary frequency interval $f \in [195.95 \text{ THz}, 196.59 \text{ THz}]$ is selected as the frequency range of interest, and the Savitzky–Golay filter is applied to the measured magnitude data. The original and filtered magnitude responses are shown in Fig. 20.

Before applying the proposed method, the magnitude response is shifted to the baseband frequency range $f \in [-320 \text{ GHz}, 320 \text{ GHz}]$. Figure 21 presents the approximation of $I(s)$, achieving a maximum absolute error below -40 dB with 60 poles. Then, the transfer function of $I(s)$ in pole-residue form is converted into its pole-zero representation. The distribution of zeros is shown in Fig. 22, where two imaginary zeros $-j2.38 \times 10^{12}$, $j2.17 \times 10^{12}$ appear outside the modeling range.

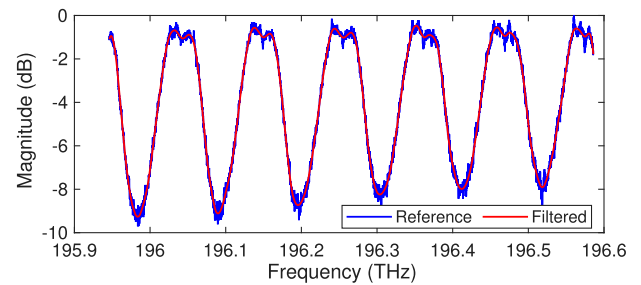


Fig. 20. Example C. Original and filtered magnitude responses of the four rings loaded MZI over the frequency range [195.95 THz, 196.59 THz].

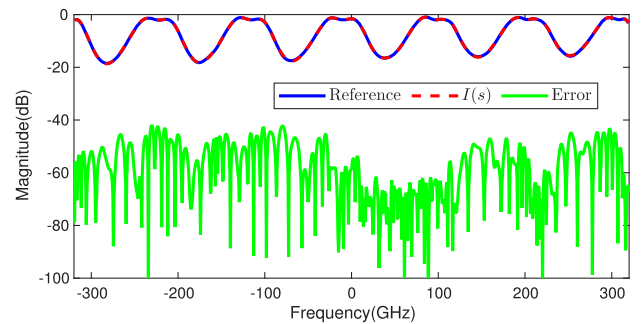


Fig. 21. Example C. The approximation results of $I(s)$.

To better illustrate the relation of imaginary zeros and the negativeness of $I(s)$, $I(s)$ over a wider frequency range is plotted in Fig. 23 using a linear scale.

Then, the two zeros are discarded to derive the transfer function $\tilde{I}(s)$. Spectral factorization is then applied to derive $\tilde{P}(s)$. Finally, the phase response of $P(s)$ is extracted from $\tilde{P}(s)$, as shown in Fig. 24.

The full frequency response is then constructed by pairing the synthesized phase with the magnitude data. Subsequently, the CVF algorithm is applied to the complete frequency response to construct a rational model using 30 poles. The resulting model achieves a maximum absolute error of -49 dB , as shown in Fig. 25. This example further demonstrates the

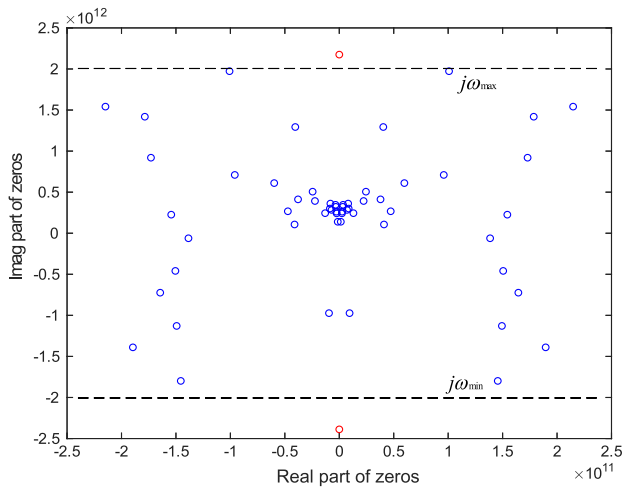


Fig. 22. Example C. The distribution of zeros of $I(s)$. The blue circles represent the zeros that are symmetric about the complex axis while the red circles indicate the imaginary zeros.

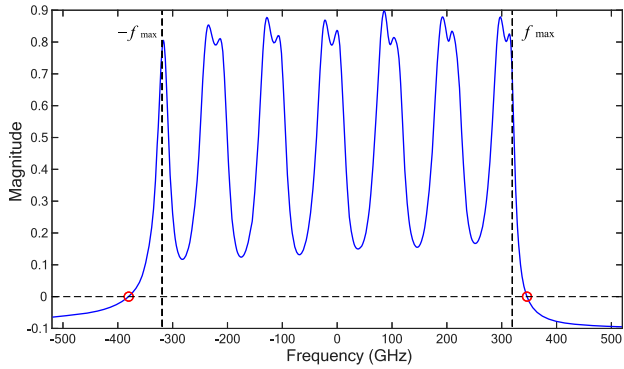


Fig. 23. Example C. Response of $I(s)$ with two imaginary zeros.

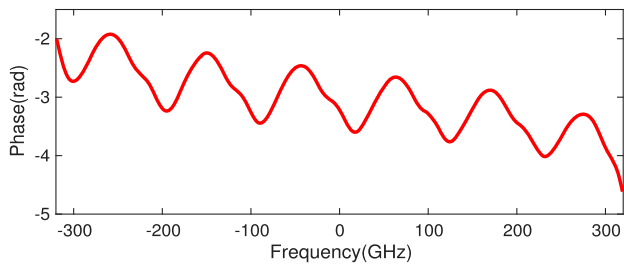


Fig. 24. Example C. The synthesized phase via the proposed method.

validity and effectiveness of the proposed method on experimentally measured data.

5. DISCUSSION

The proposed method synthesizes a physically consistent phase from magnitude-only responses, ensuring that the given magnitude and the synthesized phase satisfy the Kramers–

Kronig relations. With this physically consistent phase, the CVF technique is able to construct a rational model that accurately captures the magnitude response and provides a physically consistent synthesized phase response. In contrast, when the phase is assigned arbitrarily, the CVF technique generally fails to converge to an accurate model.

Although the resulting rational model can reproduce the same magnitude response as that of the PIC under study, the synthesized phase may differ substantially from the true physical phase, as illustrated in Figs. 10 and 17. From a theoretical standpoint, the original phase cannot be uniquely recovered from magnitude-only data. The synthesized phase obtained in this work corresponds to a near minimum-phase system, in which most zeros of the rational model are located in the left half of the complex plane. Indeed, the phase response can be readily altered by reflecting zeros from the left half-plane to the right half-plane, without affecting the magnitude response of the model or its compliance with the Kramers–Kronig relations.

A critical question therefore arises: can the given magnitude response paired with the synthesized phase still yield accurate time-domain simulation results? To address this question, a time-domain experiment is conducted using the filter in Section 4.B. The proposed method is applied to the transmission response $|S_{14}|$ to generate a synthesized phase. Figure 26 presents the magnitude response $|S_{14}|$ together with a comparison between the original phase and the synthesized phase, revealing a noticeable discrepancy between the two.

Two CVF rational models are then constructed. The first model is fitted using the original magnitude and phase, employing 30 poles and achieving a maximum absolute error of -83 dB. The second model is fitted using the same magnitude response but paired with the synthesized phase; it also uses 30 poles and attains a maximum absolute error of -80 dB. To evaluate the time-domain filtering characteristics of these two models, a sinc-shaped input signal is used as excitation. Note that a time-domain sinc signal corresponds to a rectangular spectrum in the frequency domain, with a bandwidth of approximately 320 GHz, such that high-frequency components are effectively filtered out.

Figure 27 shows the input signal and compares the output responses of the two models. It can be observed that the output generated by the model using the original phase exhibits a longer time delay than that obtained from the model with the synthesized phase. After compensating for this delay by time-shifting one of the signals, the two output waveforms are found to be identical in shape. This result suggests that, although the synthesized phase may deviate from the true physical phase, it preserves the essential dispersion characteristics within the signal bandwidth. However, it is important to note that additional caution is necessary when the constructed model is used in simulations involving wideband signals or cascaded blocks. This is because the difference between the synthesized phase and the true physical phase may lead to unexpected results. Future work will investigate cases in which the synthesized phase introduces frequency-dependent group delay variations, such as wideband excitations, strongly non-minimum-phase systems, and cascaded photonic circuits, where phase inaccuracies may lead to waveform distortion.

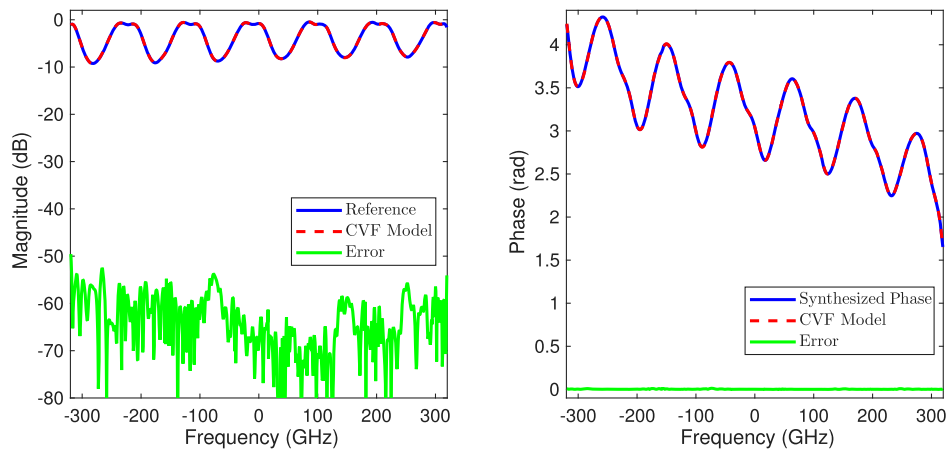


Fig. 25. Example C. The CVF modeling accuracy of the constructed full frequency response (original magnitude and synthesized phase).

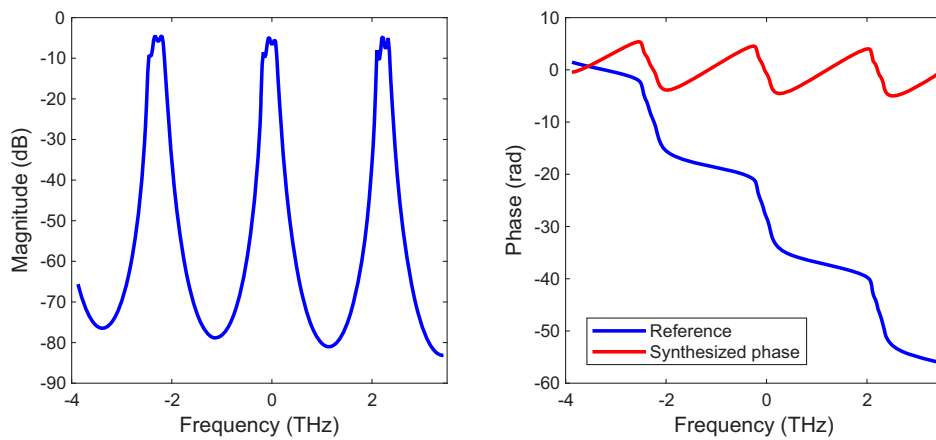


Fig. 26. Frequency response S_{14} of the five-ring resonator in Section 4.B. Left: magnitude response. Right: comparison between the original phase and the synthesized phase.

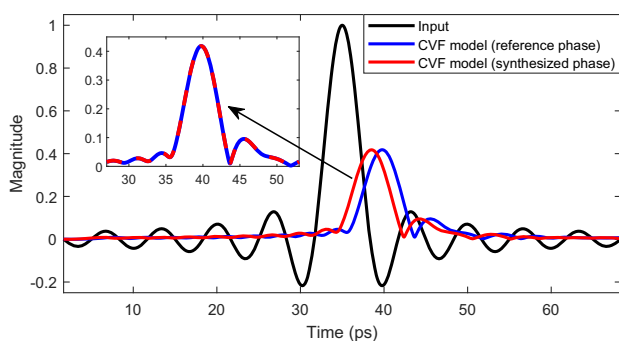


Fig. 27. Time-domain simulation comparison of CVF models using the original phase and the synthesized phase. After delay compensation, the two output waveforms are identical in shape.

6. CONCLUSION

In practical PIC measurements, phase information is typically unavailable, which has long hindered the construction of physically consistent models for circuit-level simulation. This paper introduces a robust modeling framework that overcomes

this limitation by synthesizing a physically consistent phase from magnitude-only responses through explicit handling of imaginary-axis zeros. The synthesized phase satisfies the Kramers–Kronig relations corresponding to the given magnitude response. Based on this synthesized phase, a rational model whose magnitude response matches that of the PIC under study can be constructed and directly employed for circuit-level time-domain analysis. Three representative PIC examples are presented to demonstrate the applicability and effectiveness of the proposed method.

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