Related preprint:
https://arxiv.org/abs/2508.00565

An improved Ising formulation of Max-3-Cut using higher-order spin interactions

Robbe De Prins[a,b], Guy Van der Sande[b],
Peter Bienstman[a], and Thomas Van Vaerenbergh[c]

[a] Ghent University – imec, Ghent, Belgium, [b] Vrije Universiteit Brussel, Brussels, Belgium
[c] Hewlett Packard Labs, Diegem, Belgium

Problem setting. Many combinatorial optimization problems involve multi-valued variables that are typically encoded into Ising machines (IMs) using *one-hot encoding*. A canonical example is Max-3-Cut, where the goal is to color each vertex $v \in V$ of an undirected graph while minimizing the number of edges $(uv) \in E$ between vertices of the same color. Ref.[1] previously proposed a (quadratic) Ising formulation where every vertex v is represented by a triplet of spins $\sigma_{v,c}$ ($c \in \{1,2,3\}$) using one-hot encoding: ↑↓↓ ≡ red, ↓↑↓ ≡ green, and ↓↓↑ ≡ blue. The corresponding Hamiltonian is:

$$\mathcal{H}_{\text{Ising}} = A\sum_{v\in V}\left(1-\sum_{c=1}^{3}\frac{\sigma_{v,c}+1}{2}\right)^2 + B\sum_{(uv)\in E}\sum_{c=1}^{3}\frac{\sigma_{u,c}+1}{2}\frac{\sigma_{v,c}+1}{2}. \qquad (1)$$

The A-term enforces the one-hot constraint at each vertex, while the B-term penalizes edges whose endpoints share the same color. However, even if V only contains a single vertex, Eq.1 yields a rugged energy landscape, since changing the color of the vertex requires two spin flips, thereby passing through a higher-energy spin configuration. As visualized in Fig.1(a), this ruggedness remains when the spins $\sigma_{v,c} = \pm 1$ are continuously relaxed to $s_{v,c} \in \mathbb{R}$. Fig.1(c) highlights the energetic minima, showing they are disconnected by energy barriers.

Solution. We propose a higher-order Ising formulation that counters this type of ruggedness, yielding smoother energy landscapes, both for discrete and continuous spins:

$$\mathcal{H}_{\text{HO}} = A\sum_{v\in V}\sum_{c\neq d}\sigma_{v,c}\sigma_{v,d} + B\sum_{(uv)\in E}\sum_{c\neq d}\sigma_{u,c}\sigma_{v,c}\sigma_{u,d}\sigma_{v,d}. \qquad (2)$$

Here, the number of valid colorings is doubled: ↑↓↓ ≡ ↓↑↑ ≡ red, ↓↑↓ ≡ ↑↓↑ ≡ green, and ↓↓↑ ≡ ↑↑↓ ≡ blue, allowing all colors to be reached through single spin flips. Fig.1(b,d) show that the corresponding continuous energy landscape of a single vertex contains a flat, barrier-free energy path that connects these configurations.

Evaluation. We benchmark both formulations on an analog IM for instances containing up to 800 vertices. Fig.1(e) shows that Eq.2 yields faster time-to-solution than when one-hot encoding is used. At the conference, we will also show that it yields higher success rates and lower TTS than an empirically rescaled Ising formulation proposed in prior work.

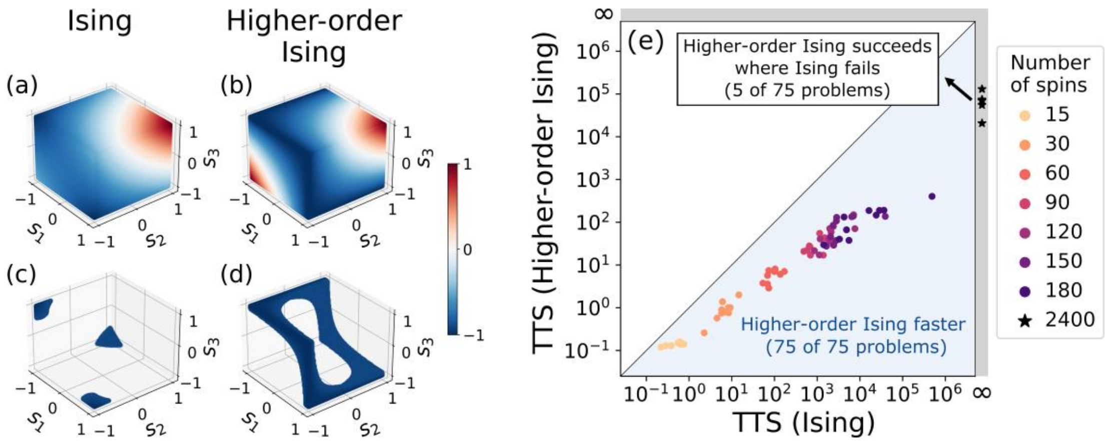

Figure 1: Normalized energy landscapes of a single spin triplet for the Ising (a,c) and higher-order Ising (b,d) formulations. Panels (c,d) highlight low-energy states (≤ -0.9). (e) Time-to-solution comparison on an analog Ising machine for Max-3-Cut instances.

[1] A. Lucas. Frontiers in physics **2** (2014), 5.