

Discretized spin interactions enhance higher-order analog Ising machines

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Context. To solve satisfiability (SAT) problems using Ising machines (IMs), higher-order spin interactions are crucial [1]. We model analog IMs using the following equation:

$$\frac{ds_i}{dt} = -s_i + \tanh(\alpha s_i + \beta I_i), \quad (1)$$

where $s_i \in \mathbb{R}$ is the i^{th} spin amplitude, α is the linear gain, β is the interaction strength, and I_i is the local field acting on s_i . For cubic spin interactions, the local field is often modeled as:

$$I_i^{\text{baseline}} = J_i^{(1)} + \sum_j J_{ij}^{(2)} s_j + \sum_{j < k} J_{ijk}^{(3)} s_j s_k. \quad (2)$$

$J_{i_1 \dots i_p}^{(p)}$ denote p^{th} -order interaction strengths, whose values are chosen such that Eq.2 encodes a given SAT problem when all spins are saturated ($s_i = 1, \forall i$). When $s_i \neq 1$, however, terms of different orders are rescaled unevenly, which may degrade the IM’s performance.

Our work. We numerically benchmark methods that aim to mitigate these imbalances:

$$I_i^{\text{spin sign}} = J_i^{(1)} + \sum_j J_{ij}^{(2)} \text{sgn}(s_j) + \sum_{j < k} J_{ijk}^{(3)} \text{sgn}(s_j) \text{sgn}(s_k), \quad (3)$$

$$I_i^{\text{rescaling}(q)} = \left(J_i^{(1)} + \sum_j J_{ij}^{(2)} \frac{s_j}{\langle |s_m| \rangle} + \sum_{j < k} J_{ijk}^{(3)} \frac{s_j s_k}{\langle |s_m| \rangle^2} \right) \langle |s_m| \rangle^{q-1}, \quad (4)$$

where $\langle |s_m| \rangle$ denotes the average absolute value of all spins. We evaluate time-to-solution (TTS) and success rate on uniform random 3-SAT and 4-SAT instances. The TTS results for 3-SAT are shown in Fig. 1, while additional results will be presented at the conference. We observe that Eq.3 generally yields faster solutions than the models of Eq.2 and Eq.4 (for $q \in \{1,2,3\}$). Eq. 3 also solves more problems within the allocated time, yielding finite TTS values where the other methods do not. This conclusion generalizes our prior work [2], which showed that spin-sign-based interactions effectively mitigate imbalances induced by external fields in quadratic analog IMs. In the higher-order setting considered here, the advantage of the spin sign method is substantially more pronounced, as it mitigates imbalances across more than two interaction orders. This work also shows that earlier conclusions drawn for the higher-order simulated bifurcation algorithm [3], which reported reduced performance of spin-sign-based interactions, apply only to purely third-order models that exclude linear and quadratic terms, and therefore exclude many industrial SAT problems. Finally, we will show at the conference that smooth approximations of the sign functions in Eq. 3 make this approach compatible with analog hardware. Our results underscore the central role of spin-sign-based interactions in enabling robust and scalable higher-order analog IM dynamics.

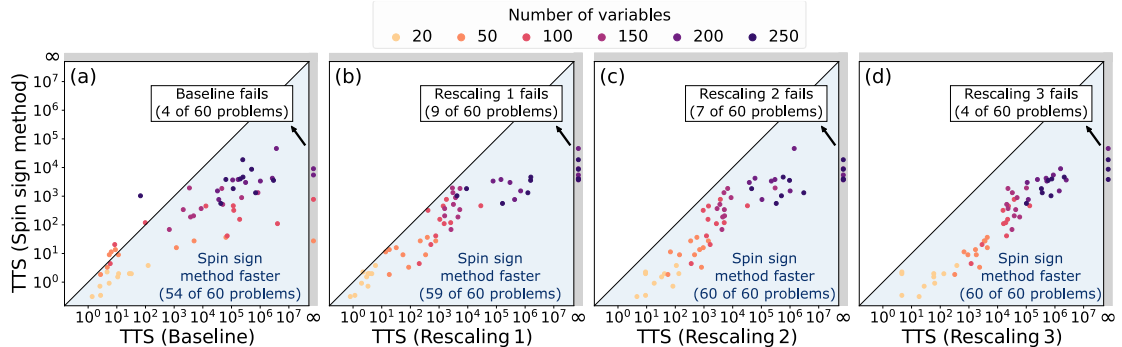


Figure 1: Comparison of time-to-solution (TTS) between the local field models of Eqs. 2-4, applied to 3-SAT problems. Instances shown in the grey area on the right were solved by the spin sign method within the allocated compute time of $t_{\text{max}}=10^4$, but not by the method on the x-axis.

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 - [2] R. De Prins, et al. *Communications Physics*. **8** (2025), 482.
 - [3] T. Kanao, et al. *Applied Physics Express*. **16** (2022), 014501.