Switchable dominant nonlinearity in coupled nonlinear silicon micro-ring resonators

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Abstract: We theoretically demonstrate spontaneous symmetry breaking in coupled silicon micro-rings, where flipping the inter-ring coupling sign enables changing the dominant nonlinearity between thermally-driven (μs-scale) or free-carrier-driven (ns-scale). When both coexist, self-pulsing emerges.

1. Introduction

Spontaneous symmetry breaking (SSB) underpins analog decision-making in coherent Ising machines (CIMs) built from degenerate optical parametric oscillators [1,2], which remain bulky and hard to scale. Integrated silicon micro-rings offer a CMOS-compatible alternative, exhibiting SSB via thermo-optic (TO) [3] or free-carrier dispersion (FCD) nonlinearities [4]. These two effects produce opposite resonance shifts (red vs. blue) on vastly different timescales (μs vs. ns). No existing coupled-ring architecture programmably selects between them or exploits their competition. Here, we show in simulation that inverting the inter-ring coupling κ sign switches the dominant nonlinearity, enabling TO-driven SSB ($\kappa < 0$) or FCD-driven SSB ($\kappa > 0$, $\approx 80\times$ faster), and we characterize the limit-cycle dynamics that emerge when they compete.

2. Model

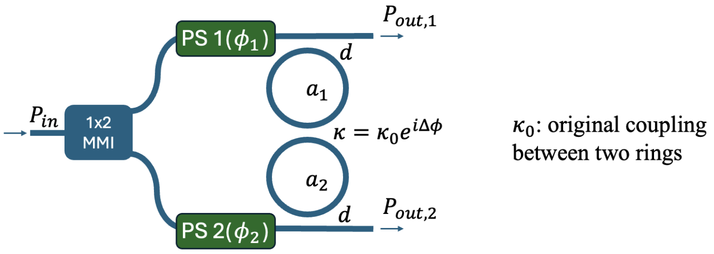

Fig. 1. Proposed SSB unit: coupled nonlinear ring pair fed by a 1×2 MMI splitter. Programmable phase shifters PS1 and PS2 can tune the effective inter-ring coupling κ.

We propose a coupled-ring-based SSB unit in (Fig. 1) where the effective inter-ring coupling κ can be tuned by phase shifters. We model the system using temporal coupled-mode theory (CMT) [4] and normalize time to the photon lifetime ($\tau = t/\tau_{\text{ph}}$):

$$\frac{da_{1,2}}{d\tau} = \left[j(-\delta - \Theta_{1,2} + n_{1,2}) - (1 + f n_{1,2})\right] a_{1,2} + j\sqrt{2p} + j\kappa a_{2,1}, \tag{1}$$

$$\frac{d\Theta_{1,2}}{d\tau} = \frac{\tau_{\text{ph}}}{\tau_{\text{th}}}\left[-\Theta_{1,2} + |a_{1,2}|^2 (1 + ef n_{1,2})\right], \tag{2}$$

$$\frac{dn_{1,2}}{d\tau} = \frac{\tau_{\text{ph}}}{\tau_{\text{fc}}}\left[-n_{1,2} + \frac{|a_{1,2}|^4}{q^2}\right], \tag{3}$$

where $a_{1,2}$ are the normalized optical amplitudes, $\Theta_{1,2}$ are the thermal detunings (TO exhibits red-shift), and $n_{1,2}$ are the free-carrier densities (FCD exhibits blue-shift). The rings interact via a coupling κ and are driven by an input power p with laser-cavity detuning δ. Using realistic silicon parameters [4], we set the TO-to-FCD power ratio q=0.397, timescale ratios $\tau_{ph}/\tau_{\text{th}}$=0.00154, $\tau_{ph}/\tau_{\text{fc}}$=0.0189, $\tau_{fc}/\tau_{\text{th}}$=0.0815, free-carrier absorption (FCA) heating coefficient e=5, FCA-to-FCD ratio f=0.0714, and coupling-to-loss ratio k=1.

3. Results

Selecting the dominant nonlinearity for SSB. By injecting triangular input signals, we find that when $\delta < 0$, TO red-shifts drive SSB once the effective detuning $\Gamma = \delta + \kappa$ crosses the threshold $\Gamma < -\sqrt{3}$, derived from coupled-cavity nonlinear dynamics [5] (Fig. 2a, FCD excluded). Inverting to $\delta > 0$ and $\kappa > 0$ flips the condition to $\Gamma > \sqrt{3}$, activating FCD blue-shift dynamics ≈80× faster (Fig. 2b, TO excluded).

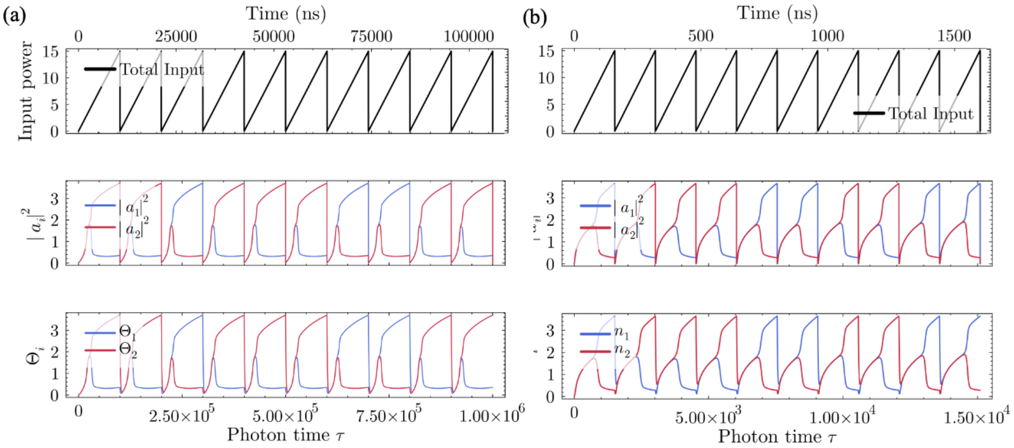

Fig. 2. Time evolution with nonlinearity-selective SSB: (a) TO-driven ($\delta = -2.5$, $\kappa = -1.0$, FCD excluded), showing SSB on the thermal timescale. (b) FCD-driven (δ=2.0, κ=2.0, TO excluded), showing ≈80× faster SSB evolution. Both cases exhibit random mirror-symmetry breaking.

TO–FCD interplay. While the previous simulations isolate these effects, in practice silicon rings exhibit both simultaneously. At $q = P_{\mathrm{th}}/P_{\mathrm{el}} = 0.397$ and $\tau_{\mathrm{fc}}/\tau_{\mathrm{th}} = 0.0815$, competing TO and FCD destabilize the steady SSB state. This produces Hopf-bifurcation-induced self-pulsing (Fig. 3a) or discrete free-carrier pulse trains (Fig. 3b). Residual thermal heating introduces memory across cycles, deterministically flipping the symmetry-broken state.

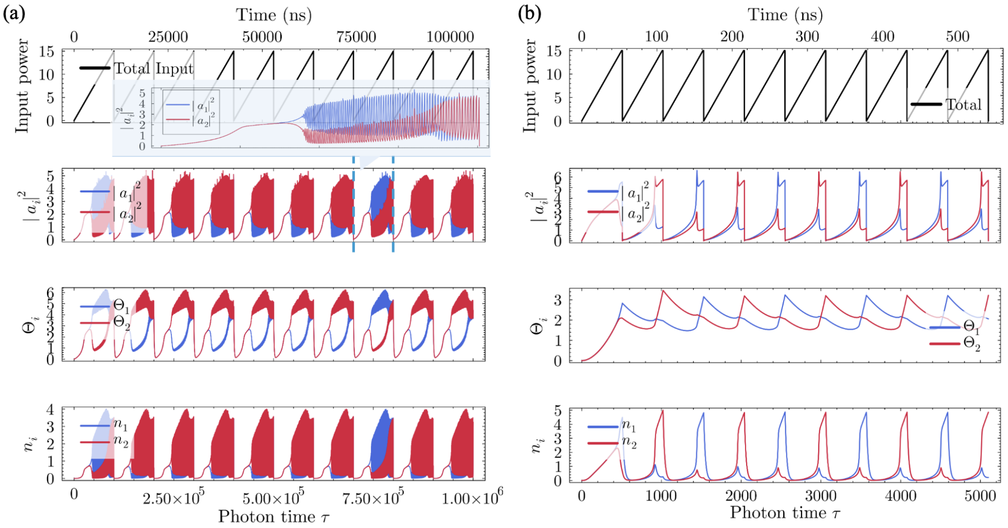

Fig. 3. TO–FCD interplay dynamics at $q = 0.397$. (a) Thermal SSB regime ($\delta = -2.5$, $\kappa = -1.0$) showing Hopf-born pulsing (see inset). (b) FCD regime ($\delta = 2.0$, $\kappa = 2.0$) showing discrete intensity pulses, where thermal memory deterministically flips the successive active states.

4. Conclusions

Flipping κ and δ in coupled silicon micro-rings selects TO-driven (μs) or FCD-driven (ns) SSB. When both nonlinearities coexist under realistic parameters, the system transitions to self-pulsing limit cycles, a regime whose implications for circuit-level integration warrant further investigation.

References

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