Coherent frequency conversion for quantum information processing

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ABSTRACT

Efficient frequency conversion is crucial for interfacing photons with a variety of quantum systems ranging from matter qubits (color centers, quantum dots, atoms), optical fibers, detectors, that often operate at widely different wavelengths. Frequency conversion is also a resource to process quantum information.[1,2] Parametric processes such as sum frequency generation (SFG) and Bragg scattering four wave mixing (BS-FWM) offer a good versatility for such frequency conversion. It is well understood that those nonlinear processes have to be strong enough and satisfy phase matching to achieve a high conversion efficiency. While it is enough to consider those two aspects for moderate conversion efficiency up to 50%, the analysis is a bit more complicated when targeting efficiencies closely approaching unity. We are reviewing the pitfall that must be avoided to indeed reach near unity frequency transduction.

Keywords: nonlinear optics, four-wave mixing, sum frequency generation

1. INTRODUCTION

Frequency conversion of single photons has been first investigated using sum frequency generation in order to make the detection of telecom (1-3-1.6 μm) photons possible with silicon avalanche photodiodes.[3,4] Several demonstrations have showed that near unity efficient conversion could be realized. With extra care, that conversion could also be made without introducing significant photon noise. A common limitation of conversion via sum frequency generation is the typical narrow bandwidth over which photons can be converted. In essence, this limitation originates from the large wavelength span separating up converted photons and incident/pump wavelengths. This limitation is precisely what can easily be relaxed using a four-wave mixing process rather than a three wave one. On on side, there exists wavelength configurations requiring a smaller frequency span in that case than in the case of a three wave mixing. On the other side, the fact that there are four photons involved makes sharing the momentum evenly easier. Efficient conversion of single photons via Bragg scattering four-wave mixing (BS-FWM) was reported as early as 2010[5] and was followed by a wide variety of implementations using optical fibres,[6–9] micro cavities[10] and attempts using nanophotonic waveguides.[11] However, in most cases the efficiency was not so close to unity but was rather in the range of 30 to 60%. This was sometimes due to an insufficient nonlinearity, or an unsatisfied phase matching condition but in some cases other explanation could be find that I will review hereafter. Indeed, competing processes can send the single photon to an untargeted frequency, they can deplete the pump beams or they even induce extra losses. Those spurious nonlinear processes are also the reason why FWM-based frequency converters typically don't behave as good as SFG-based converters with regards to their noise properties. To understand correctly the limitations in term of noise and

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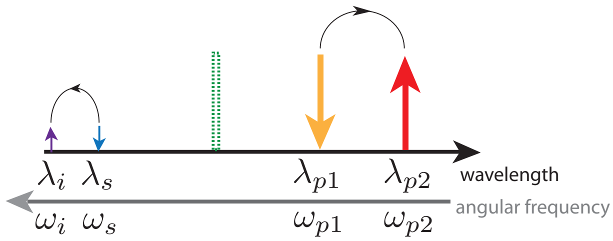

Figure 1. Frequency up-conversion via Bragg scattering four wave mixing. Up-arrows indicate the creation of a photon and down-arrows indicate the annihilation of a photon. Thick arrows represent strong fields while small ones refer to single photons.

inefficiency, it is important to know the requirement in term of optical pump power, propagation length and group velocity dispersion required for BS-FWM. The theory for BS-FWM is well known.[12] While asymmetric pumping can be of high value[13,14] it is also much more difficult to analyze[15] in a general way. Here, I consider the simplest case where the two pump involved have the same power $P_{p1} = P_{p2} \equiv P_0$ so that no nonlinear phase shift must be accounted for. The conversion "efficiency" is

$$\eta = \left| \frac{i\kappa}{\sqrt{|\kappa|^2 + |\delta k_{p1,s;p2,i}|^2}} \sin\left(\sqrt{|\kappa|^2 + |\delta k_{p1,s;p2,i}|^2} L\right)\right|^2 \tag{1}$$

where the phase missmatch $\delta k_{p1,s;p2,i} = k(\omega_{p_1}) + k(\omega_s) - k(\omega_{p2}) - k(\omega_i)$, and the nonlinear coupling $\kappa = 2\gamma P_0 \exp(i\theta)$ with γ the nonlinear parameter in watt/m^2, θ a relative phase term between the pump beams that we can neglect if the conversion occurs in a continuous single nonlinear device. Ideally, unity efficiency is possible if $\delta k_{p1,s;p2,i} = 0$ and $\kappa L = \pi/2$. If we choose a propagation $L = \frac{\pi}{2\kappa}$, then a first condition to achieve efficient conversion can be expressed as

$$\delta k_{p1,s;p2,i} \leq \kappa/10 \tag{2}$$

that ensures (in this first approximation) a conversion efficiency η greater than 99%. This is often achieved by coinciding the central frequency $\omega_0 = \frac{\omega_i + \omega_{p1}}{2}$ with the zero group velocity of the nonlinear device ω_{0-GVD}. The phase mismatch is however not necessary minimal at this particular point once higher order dispersion terms are accounted for. This appears clearly from the explicit expression of $\delta k_{p1,s;p2,i}$ we can get by making an expansion using $\beta_2 = \frac{\partial k^2}{\partial^2 \omega}$ and higher order dispersion terms $\beta_n = \frac{\partial k^n}{\partial^n \omega}$:

$$\delta k_{p1,s;p2,i} = \beta_{2;p1,s,p2,i}(\delta\omega^2 + \delta\omega\Delta\omega) + \frac{\beta_{4;p1,s,p2,i}}{12}\left(\delta\omega^4 + \delta\omega\Delta\omega(\frac{3}{2}\delta\omega\Delta\omega + \frac{\Delta\omega^2}{2} + 2\delta\omega^2)\right) \tag{3}$$

where $\delta\omega = \omega_{p1} - \omega_{p2} = \omega_i - \omega_s$ is the frequency kick and $\Delta\omega = \omega_i - \omega_{p1}$ is the spectral separation between the pump frequency channels and the single photon frequency channels and where the dispersion terms are defined at the central frequency ω_0. In optical silica fibers (see figure 2), the β_4 term in typically negative at the 0-group velocity frequency so that the β_4 and β_2 terms can cancel each other in the vicinity (at slighly higher frequency) of the 0-GVD frequency. As a result, the central frequency ω_0 should be slightly increased to achieve the best phase matching possible. The situation differs in other types of waveguides.[16]

2. EFFICIENCY LIMITATIONS

2.1 Competing BS-FWM

The most immediate competing processes to the frequency conversion from ω_s to ω_p are actually other BS-FWM processes. Figure 3 illustrates the two major spurious processes that will occur if the associated phase mismatch $\delta k_{p2,s;p1,s-}$ and $\delta k_{p1,i;p2,i+}$ respectively are sufficiently low. To avoid a reduction in conversion efficiency by less than 1%, we should therefore add to eq. 2 the following two conditions

$$\delta k_{p2,s;p1,s-} \geq 10\kappa \quad \text{and} \quad \delta k_{p1,i;p2,i+} \geq 10\kappa. \tag{4}$$

This indicates that having a very large value of κ is not desirable and rather the priority should be to achieve a phase mismatch $\delta k_{p1,s;p2,i}$ as low as possible (we have seen that its cancellation was possible).

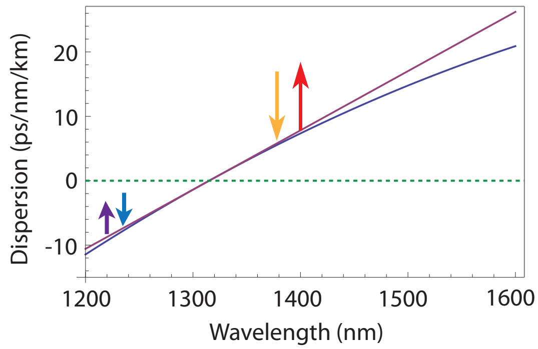

Figure 2. In waveguides, the group velocity dispersion shows a significant variation characterized by β_3 (purple curve) as well as a β_4 (blue curve) terms. In the case of a SFM-28, typical key values are $\lambda_0 = 1315$ nm, $\beta_{4,\lambda_0} = -2.2\times10^{-4}\mathrm{ps}^4/\mathrm{km}$. The arrows indicate a typical configuration for Bragg scattering four wave mixing.

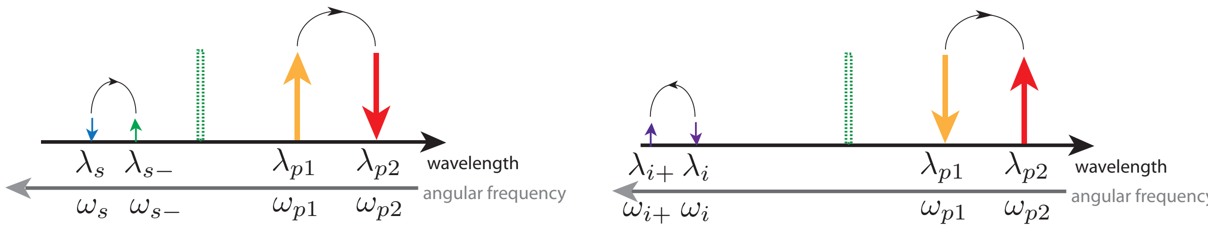

Figure 3. Two other BS-FWM processes can effectively reduce the conversion efficiency. Left: the initial photon can be down-converted rather than up-converted. Right: the up-converted photon can be up-converted a second time to reach an even higher frequency.

2.2 Bandwidth

The single photon that needs to be converted displays a finite bandwidth $\delta\omega_{bw}$. This implies that the cancellation of $\delta k_{p1,s;p2,i}$ cannot occur for the whole photon spectral wavefunction in practice. This effects strongly mitigates the conclusion made in the previous section that minimizing $\delta k_{p1,s;p2,i}$ should be a priority. The reality is that the four wavelength involved and the bandwidth $\delta\omega_{bw}$ have to be chosen as a tradeoff so that eq. 4 are satisfied as well as

$$\delta k_{p1,s+\delta\omega_{bw};p2,i+\delta\omega_{bw}} \leq \kappa/10 \quad \text{and} \quad \delta k_{p1,s-\delta\omega_{bw};p2,i-\delta\omega_{bw}} \leq \kappa/10 \tag{5}$$

Previous reports have shown this was still possible experimentally.[6]

2.3 Pump depletion

Any of the two pump $\{\omega_{p1}, \omega_{p2}\}$ can also be involved in a degenerated four wave mixing process where the other pump serves as a strong seed for the process. Given than the length L is set so that $\kappa L = \pi/2$, the gain of the degenerate process can reach -1 dB which is sufficient to deplete slightly the pump beams. To avoid this, two additional conditions can be made on the phase mismatch for those processes that is $\delta k_{p_1,p_1;p_2,p_1}$ and $\delta k_{p_2,p_2;p_1,p_2+}$ should be greater than $2\pi L$. Those conditions can be expressed in term of the group velocity dispersion at the pump frequencies :

$$\beta_{2,\omega_{p1}}\delta\omega^2 L \geq 2\epsilon\pi \quad \text{and } \beta_{2,\omega_{p2}}\delta\omega^2 L \geq 2\epsilon\pi \tag{6}$$

where $\epsilon = 1$ reflects the first destructive interference of the degenerate four wave mixing process that is the first zero of the characteristic sinc function. Taking $\epsilon = 2$ ensures a pump depletion lesser than 1%. This sets therefore an extra constraint on the dispersion and length of the nonlinear device.

Two other mechanisms are likely to deplete any of the pump beams. In fibers, the Brillouin scattering is well known to reduce power transmission of narrowband (MHz) pulses due to the large backscattering it induces.[17] While the effect becomes irrelevant for relatively short (ns) pulses, the frequency translation of high coherence

(MHz range) photons while preserving their linewidth is therefore compromised in silica optical fibers (unless structural measures are taken to damp the Brillouin process). This effect is much more limited in nanophotonic waveguide due to the damping of acoustic waves in the substrate.
Raman scattering is another process that cannot only deplete the pump power of the highest frequency pump but also amplify the lowest frequency pump. The created power asymmetry between the two pumps not only reduces κ and therefore the conversion efficiency η but it also induces a nonlinear phase susceptible to increase the phase mismatch. The Raman gain varies significantly between materials. The differences arise both in term of strength as in term of frequency of the Raman resonance. The obvious solution to this problem is therefore to take advantage of the resonant nature of the Raman scattering and chose $\delta\omega$ significantly different than this resonance. In fibers, the interplay between Kerr and Raman nonlinearity has been studied in many occasions .[18]

2.4 Linear effets

There exist many sources of linear losses that can affect the four frequencies involved. When the frequencies involved are widely spaced from each other, linear losses (absorption, bending, ...) can differ significantly between ω_i and ω_{p2}. Similarly, a waveguide may become spatially multimode at large frequencies and mode coupling may arise thus leaking single photons to an undesired mode. Mode coupling (spatial or polarization) can also affect the effective index and therefore the phase matching condition .[19]

3. NOISE

3.1 Spontaneous four-wave mixing

Spontaneous four-wave mixing (fig. 4) is well known for its use for generating pairs of correlated photons. It annihilates two photons from either of the pump beams at $\{\omega_{p1}, \omega_{p2}\}$ to create two new ones at frequencies $\{\omega_{p1} \pm \Omega, \omega_{p2} \pm \Omega, \frac{\omega_{p1}+\omega_{p12}}{2} \pm \Omega\}$ if the two annihilated photons originate respectively from the first pump, from the second pump, or one of each pump. In addition to the conservation of energy we just expressed, the process also must satisfy the conservation of momentum (a.k.a phase matching) expressed by the phase mismatch $\delta k_{p1,p1,p1+\Omega,p1-\Omega} = 2k(\omega_{p1}) - k(\omega_{p1} + \Omega) - k(\omega_{p1} - \Omega)$ for the case of degenerated four-wave mixing induced by the first pump. It is well known that the phase matching condition constraints in practice the value Ω over which photon pairs are generated. It would therefore be tempting to assume the disperse properties of the nonlinear device (crystal, waveguide, cavities) may prohibit the generation of photons at a frequency $\omega_p + \Omega$ that equals any of the single photon frequency $\{\omega_i, \omega_s\}$. The spectral response of the spontaneous four wave mixing process is actually given by the simple following relation

$$f = (\gamma PL)^2 \mathrm{sinc}^2(L\delta k_{p1,p1,p1+\Omega,p1-\Omega}) \tag{7}$$

In the simplest case, the phase mismatch is given as function of the group velocity dispersion $\beta_{2,p1}$ and the frequency detuning Ω as $\Delta k = \beta_{2,p1}\Omega^2/2$ so that the flux of photons generated decreases as the fourth power of the frequency detuning Ω. We can see in figure 4 that once Ω is ten times the detuning corresponding to the first zero of the sinc function, the photon flux is on the order of 10^{-5}. While this number seems low, this flux of parasitic photons has to be integrated over the duration of the pump pulse and over the spectrum collected in the idler and signal channels. This can increase the flux by many orders of magnitude. The photon flux spectrum depicted in figure 4 corresponds to an idealized nonlinear device where the dispersive properties are constant. In waveguides, the group velocity dispersion may change along the propagation owing to variation in its width or height. This also alters the spectrum of this photon flux.[20]

Finally, it should be noted that degenerate four wave mixing can have a very different spectral response if the waveguide is birefringent. Indeed, vectorial four wave mixing is known to display phase maxing sidebands that can be well separated from the pump.[21]

3.2 Cascaded four-wave mixing

We have already mentioned (eq. 6) that the group velocity dispersion at the pump frequencies should be sufficient to avoid pump depletion via degenerate four wave mixing. If neglected, this effect can have dramatic results on the noise properties at the single photon wavelength. Another way that degenerate four wave mixing can induce

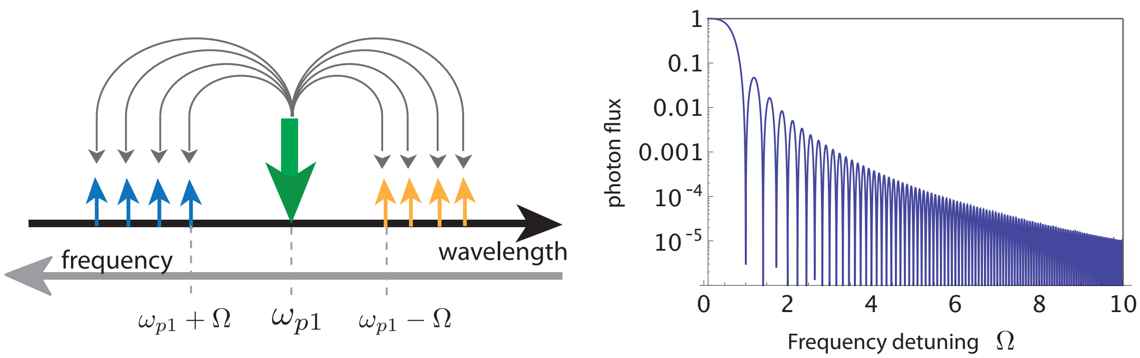

Figure 4. Left: the process of spontaneous four-wave mixing (SpFWM) generates pairs of frequency correlated photons on a broad spectral band. Right: The evolution of the spontaneous four wave mixing as the frequency detuning Ω follows a Ω^{-4} law. The process here illustrated is degenerated but SpFWM involving both pump beams is also occurring.

noise at the photon frequencies $\{\omega_s, \omega_i\}$ is by cascaded stimulated four-wave mixing (see figure 5). In the first case (left figure), the wave at ω_{p2} serves as a seed for the degenerate four wave mixing process at ω_{p1}. If the recommendation from eq. 6 is followed, the new wave at λ_{p-} can still be nearly 1% of the pump. In this situation, it is not unluckily that the group velocity dispersion at the new frequency ω_{p-} is going to be much lower and therefore the process of degenerate four wave mixing initiated by this new wave will be phase matched. The process being quadratic, the conversion of the second pump at λ_{p2} will be lesser than 10^{-4} but given the large intensity of that beam as compared to the stream of single photon, this process can have a dramatic impact. A similar process (right panel of figure 5) arises when the newly generated wave at ω_{p+} serves as a seed for another degenerate four-wave mixing process initiated by the pump at λ_{p1}. The strength of this process is of the same order as the previously described one. Higher other cascaded term are several order less important but can still induce a significant amount of photon noise.

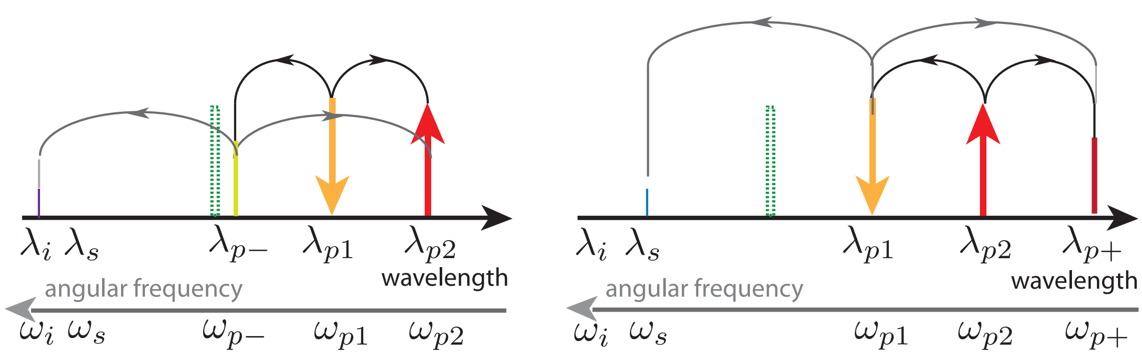

Figure 5. Two most important cascaded stimulated four-wave mixing that may add photons to the quantum channels.

3.3 Raman scattering

Raman scattering process can always add photon noise to the quantum channels. For Raman scattering, the frequency $\Delta\omega$ and $\Delta\omega + \delta\omega$ can often be adjusted to be away from the exact Raman resonant frequencies. This is particularly true in crystalline media (Si, SiN) having narrowband spectral response but can be more difficult in amorphous materials such as silica (SiO_2). If the quantum channels are at higher frequency than the pump channels (as illustrated in all the present diagrams), Raman noise can be significantly reduced by cooling. It should be pointed that other processes behaving similar to Raman scattering can produce a background of similar magnitude but over a continuous (non resonant) range in the vicinity of the pump.[22–24]

4. CONCLUSION

Many reports[10,25,26] have recently been made on efficient BS-FWM for efficient conversion. However, realization displaying both efficiency in excess of 90% and low noise are scarce[6,8,9] or even nonexistent when considering also the linear losses. Here, we have reviewed all the tricks and tips to obtain efficient and low nose frequency conversion via Bragg scattering four wave mixing. It shows that tradeoffs have to be reached and it hints at why realizations using nanophotonic waveguides have not yet reach very high efficiency: the accumulated dispersion at the pump frequencies is simply insufficient to avoid some of the spurious processes I have listed here above. By increasing the accumulated dispersion (typically using longer waveguides), there is no doubt that nanophotonic waeguides can do as good if not better than optical fibers.

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