# Comparative Assessment of Time-Domain Models of Nonlinear Optical Propagation

**Khai Q. Le[1,2*], Harshana G. Dantanarayana[2], Elena A. Romanova[3], Trevor Benson[2], and Peter Bienstman[1]**

*[1]Photonics Research Group, Department of Information Technology, Ghent University, B-9000 Ghent, Belgium*

*[2] George Green Institute for Electromagnetics Research, University of Nottingham, NG7 2RD Nottingham, UK*

*[3] Saratov State University, Astrakhanskaya 83, 410012 Saratov, Russia*

*E-mail: khai.le@intec.ugent.be*

**ABSTRACT**

We present a comparative assessment of time-domain approaches for modelling nonlinear optical propagation, focussing on a finite-difference time-domain beam propagation method (TD-BPM) and the rigorous transmission line modelling (TLM) method. The assessment is carried out on the basis of reflection and transmission of non-stationary light beams propagating through the junction of linear and nonlinear waveguides.

## 1. INTRODUCTION

The availability of laser sources generating high-intensity femtosecond optical pulses has recently inspired tremendous research interest in the study and elaboration of novel guiding structures and materials for nonlinear optics applications. In order to study the spatiotemporal dynamics of femtosecond laser pulses propagating in a Kerr-type nonlinear medium, various treatments have been proposed including those based on the generalized nonlinear Schrodinger equation (GNLSE), the finite-difference time-domain (FDTD) method and the transmission line modelling (TLM) method [1].

A GNLSE-based method is usually feasible when modelling the propagation of optical pulses whose duration is more than several periods of oscillations of the carrier frequency. However, the main limitation of the method lies in the paraxial approximation to the wave equation under the slowly varying envelope approximation (SVEA) where the second-order derivatives in the wave equation are ignored.

The FDTD and TLM methods are well-known rigorous time-domain techniques providing reliable conduits for comparisons. The main difference between these two widely used time-domain techniques is the layout of the time-stepping and the unit cell process. In the TLM method, the fields are solved at the same time instant at the centre of the TLM cell resulting in a straightforward solution of nonlinear equations, whereas in the FDTD method there is a separation of half a space step and half a time step between the electric and magnetic fields. However, these methods require a very small time step size to fulfil the stability criterion; this leads to a substantial increase in the computation resources required, especially for the analysis of long lengths of optical waveguides.

Recently, simple and efficient BPMs in the time domain have been developed [2,3]. These deal with reflected waves, and have been successfully employed for the analysis of both TE and TM-modes propagating in photonic crystal structures. The time-domain BPMs allow higher time step size than TLM and FDTD (thus resulting in reduced computational efforts) and achieve high-order accuracy not only in space but also in time. In this paper, we present a comparative assessment between time-domain BPMs and the other existing time-domain methods by investigating the reflection and transmission of the non-stationary light beams propagating through the junction of linear and nonlinear waveguides.

## 2. WB-FDTD-BPM

A detailed description of GNLSE-based and TLM methods was already presented in [1]. For time-domain BPM, we start with its formulation by considering a two-dimensional (2D) scalar wave equation where the computational domain lies in the xz-plane:

$$\frac{\partial^2 \Psi}{\partial z^2} + \frac{\partial^2 \Psi}{\partial x^2} = \frac{n^2}{c^2}\frac{\partial^2 \Psi}{\partial t^2}\,. \qquad (1)$$

Here n is a refractive index profile and c is the speed of light in free space. The formal solution of Eq. (1) with a slowly varying complex amplitude is given by

$$\Psi(x,z,t) = \psi(x,z,t)\exp(i\omega_0 t)\,. \qquad (2)$$

The work reported in this paper was supported in part by the short term scientific mission (STSM) program of COST Action MP0702 and in part by the funds from the Photonics@Be project and a Royal Society Joint Project.

We obviously only consider waves propagating forward in time; the method is fully reflective since no approximation is made in the propagation direction. By substituting Eq. (2) into Eq. (1), we obtain

$$\frac{\partial^2 \psi}{\partial t^2} + 2i\omega_0 \frac{\partial \psi}{\partial t} = P\psi \ , \tag{3}$$

where $P = \frac{c^2}{n^2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}\right) + \omega_0^2$ and $\omega_0$ is the centre angular frequency. From this equation, its wide-band solution can be obtained by various approximate approaches. Among those, Padé approximant operators have become one of the most commonly used techniques. Recently, we have demonstrated that the use of the modified Padé approximant operators can offer the advantage of using larger propagation steps than the conventional Padé operators for the same accuracy, with an associated reduction in computational efforts. For more details on this approach, we refer to [4]. Here the TD-BPM employed is based on a Padé(1,1) approximant operator.

For propagation in a material having instantaneous Kerr nonlinearity, the refractive index can be expressed as

$$n = n(x, z, t; |\psi|^2) \ . \tag{4}$$

Since it depends on the intensity of the field, an iterative algorithm is included for efficiently evaluating the nonlinear refractive index. The solution at the forward step is recalculated with the modified nonlinear refractive index, and the scheme is continued until the solution at the forward step converges.

## 3. Results and discussion

In this section, we perform a comparative assessment of the time-domain methods mentioned above by the simulation of the excitation of a nonlinear planar waveguide by its linear mode. The structure examined is shown in Fig. 1 where the linear and nonlinear waveguides of the junction have the same core thickness *d* and linear refractive index profile. The problem is reduced to the consideration of a TE-polarized non-stationary light beam and instantaneous Kerr nonlinearity. The initial spatiotemporal distribution in the linear waveguide is taken as a Gaussian pulse in time and a linear fundamental mode in the transverse plane at wavelength $\lambda_0 = 1.53$ μm.

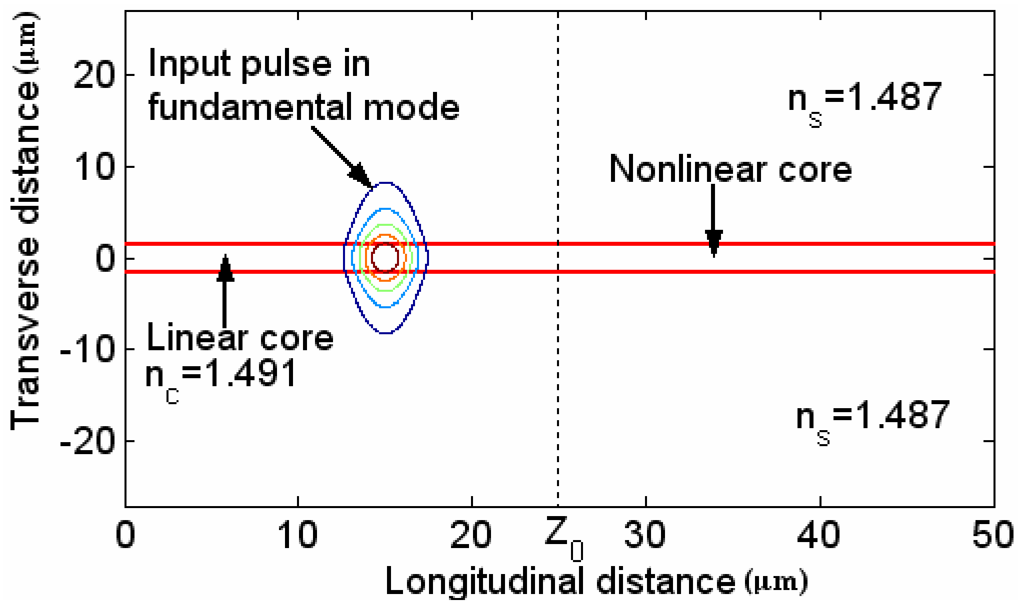


*Figure 1. Junction of linear and nonlinear planar waveguides.*

As a reference parameter for the nonlinear propagation, we use $n_K I_0$, where $I_0$ is the peak intensity of the input pulse and $n_K$ is Kerr nonlinear coefficient. This parameter is dimensionless and combines both nonlinear material properties and the highest beam intensity. It determines the maximum value of the nonlinear part of the real refractive index induced by the high-intensity light beam in the nonlinear material with Kerr nonlinearity.

In the comparative assessment, we only consider short propagation distances in the nonlinear waveguide and material dispersion effects are ignored. The non-stationary light beam propagating is simulated by employing a uniform transverse computational grid over the space ($-X_2$:$X_2$) so that the spatial distribution of the electric field at a given moment is calculated. As an output parameter the normalized energy of the pulse propagating in the nonlinear waveguide is evaluated as:

$$R_n(t), T_n(t) = \frac{\int_{Z_1}^{Z_2} \int_{-X_n}^{X_n} dzdx \left|\psi(x,z,t)\right|^2}{\int_{0}^{\infty} \int_{-X_2}^{X_2} dzdx \left|\psi(x,z,0)\right|^2} \text{; n=1,2} \quad (5)$$

where for reflection ($R_n$) $Z_1=0$ and $Z_2=Z_0$ and for transmission ($T_n$) $Z_1=Z_0$ and $Z_2=\infty$. The transverse integration is taken over the waveguide core ($x<|X_1|$), or the full range ($x<|X_2|$) within the moving computational frame.

In this work, results are calculated for two pulse durations (18 and 36 fs) and are shown in Figs. 2 and 3. The pulse is launched in the linear waveguide at time t=0, and centred 10 μm away from the linear and nonlinear boundary. The reflected and transmitted energies are evaluated when the transmitted pulse just resides fully in the nonlinear waveguide.

From the numerical simulations (results not presented here), we observed that low intensity pulses propagate in the cladding at different angles to the waveguide axis in the backward and forward directions. Some part of the reflected radiation is confined in the linear waveguide and propagates in the backward direction in the core, the pulse duration being less than that of the initial pulse. The reflection coefficient calculated (the total normalized energy reflected by the junction $R_2$) increases with $n_K I_0$ and does not depend on pulse duration within each method as seen in Fig. 2a. The set of results calculated by WB-FDTD-BPM and TLM show a slight difference because of the low order Padé(1,1) approximant operator of the TD-BPM. The agreement between results could be improved by using higher order Padé approximant operators [2]. The distribution of the reflected radiation in the core depends on both $n_K I_0$ and the pulse duration, as shown in Fig. 2b. The slight dependency in this reflection coefficient on pulse duration is due to the fact that with an increase in pulse duration a greater part of the initial pulse energy is scattered into the cladding.

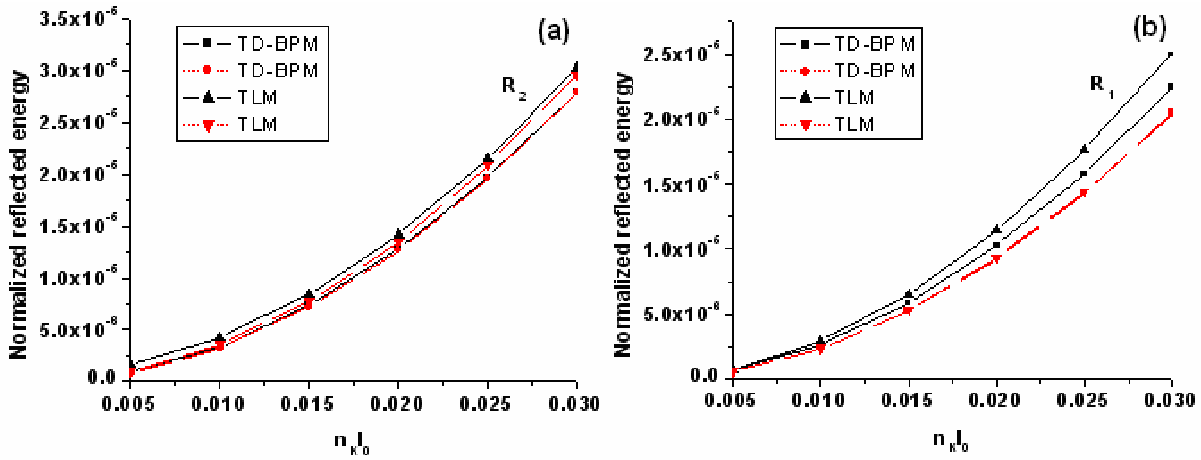


*Figure 2. (a) Total reflected energy $R_2(t)$ and (b) reflected energy in the core $R_1(t)$. Pulse durations 18 fs (solid lines) and 36 fs (dashed lines).*

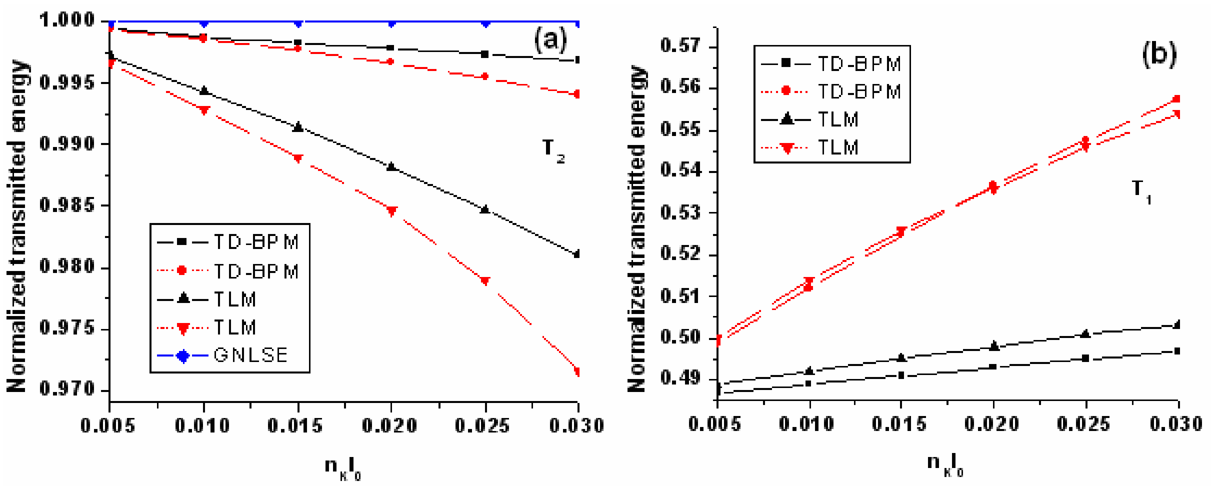


*Figure 3. (a) Total transmitted energy and (b) transmitted energy in the core. Pulse durations 18 fs (solid lines) and 36 fs (dashed lines)*

The energy $T_1$ of the forward propagating pulse in the core of the nonlinear waveguide increases with an increase in $n_K I_0$ due to the self-focusing effect (Fig. 3a). This is accompanied by a decrease of the total transmitted energy $T_2$ due to radiative losses, as seen in Fig. 3b. For a given value of $n_K I_0$ the normalized transmitted energy in the core decreases with an increase in initial pulse duration. This may be explained by the fact that there is a difference in the initial pulses' energy. In Fig. 3 the dashed lines correspond to a pulse duration twice that of the solid lines; this means a two-fold increase of the initial energy. In general, the TLM results show greater spatial variations of the light beam in the process of self-focusing in comparison with the TD-BPM results. There exists a small difference between the transmitted energies calculated by the FDTD-BPM and TLM methods. This, we believe, is due to the low order approximation in bandwidth of the Padé(1,1) FDTD-BPM method used. The GNLSE scheme used is not a reflective one and total forward propagating energy remains constant.

## 4. CONCLUSIONS

In this paper, numerical time-domain techniques have been employed to study the excitation of a nonlinear planar waveguide by a non-stationary light beam. TLM is already established as a rigorous technique to model transmitted and reflected beams.. However, it requires a very small time step (Courant condition) and thus significant computational effort is needed. The FDTD-BPM provides an attractive alternative numerical technique for nonlinear optical analysis and even with a Padé(1,1) approximation in time gives a good improvement over the GNLSE-based approach.

## ACKNOWLEDGEMENTS

The work reported in this paper was supported in part by the short term scientific mission (STSM) program of COST Action MP0702 and in part by the funds from the Photonics@Be project and a Royal Society Joint Project.

## REFERENCES

[1] E. A. Romanova, V. Janyani, A. Vukovic, P. Sewell, and T. M. Benson: Models of nonlinear waveguide excitation by non-stationary light beam, *Opt. Quantum. Electron., vol. 39, pp. 813-823, 2007.*

[2] J. Shibayama, A. Yamahira, T. Mugita, J. Yamauchi, and H. Nakano: A finite-difference time-domain beam-propagation method for TE- and TM-wave analyses, *J. Lightwave Technol., vol. 21, pp. 1709-1715, 2003.*

[3] T. Fujisawa, and M. Koshiba: Time-domain beam propagation method for nonlinear optical propagation analysis and its application to photonic crystal circuits, *J. Lightwave Technol., vol. 22, pp. 684-691, 2004.*

[4] Khai Q. Le, T. M. Benson, and P. Bienstman: Application of the modified Padé approximant operators for time-domain beam propagation method, *accepted for publication in J. Opt. Soc. Am. B.*