Wide-angle beam propagation method without using slowly varying envelope approximation

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A new wide-angle (WA) beam propagation method (BPM) is developed whereby the exact scalar Helmholtz propagator is replaced by any one of a sequence of higher-order (m,n) Padé approximant operators. Unlike the previous well-known WA-BPM proposed by Hadley [Opt. Lett. **17**, 1426 (1992)], the resulting formulations allow one a direct solution of the second-order scalar wave equation without having to make slowly varying envelope approximations so that the WA formulations are completely general. The accuracy and improvement of this approximate calculation of the propagator is demonstrated in comparison with the exact result and existing approximate approaches. The method is employed to simulate two-dimensional (2D) and three-dimensional (3D) optical waveguides and compared with the results obtained by the existing approach. © 2009 Optical Society of America

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1. INTRODUCTION

Efforts to improve the limitations of the paraxial approximation or Fresnel equation in the beam propagation method have so far made use of wide-angle (WA) formulations. Different treatments of the wide-angle beam propagation method (WA-BPM) based on the slowly varying envelope approximation (SVEA) have been developed. In these approaches the field is assumedly separated into two parts including the complex field amplitude and a propagation factor [1]. There exist rational approximants of the square root operator, the exponential of the square root operator [2], the real Padé approximant operators mentioned here as Hadley(m,n) [3], and the complex Padé approximant operators [4] for rectangular coordinates as well as an oblique coordinate system [5]. In addition, treatments of WA-BPM without having to make the SVEAs have also been reported, including the series expansion technique of the propagator [6], the rational approximation of the one way propagator [7], and the split-step of the beam propagation equation [8,9]. The Padé-approximant-based WA-BPM is one of the most commonly used techniques for modeling optical waveguide structures. It is a nonlinear expression in the form of a rational function $(N(m)/D(n))$, a ratio of two polynomials that are given by recurrence equations [3,4]. By sharing the same idea without having to make the SVEA, we present a new WA-BPM whereby the exact scalar Helmholtz propagation operator is approximated by any one of a sequence of higher-order (m,n) Padé approximant operators.

2. FORMULATION

A. Padé Approximant Operators for WA-BPM

The scalar Helmholtz equation is given by [3]

$$\frac{\partial^2\Psi}{\partial x^2}+\frac{\partial^2\Psi}{\partial y^2}+\frac{\partial^2\Psi}{\partial z^2}+k_0^2n^2(x,y,z)\Psi=0, \tag{1}$$

or

$$\frac{\partial^2\Psi}{\partial z^2}=-P\Psi, \tag{2}$$

where $P=\nabla_\perp^2+k_0^2n^2=(\partial^2/\partial x^2)+(\partial^2/\partial y^2)+k_0^2n^2$ with n as the refractive index profile and k_0 as the vacuum wave vector. Note that there is no reference refractive index included in operator P.

By multiplying both sides of Eq. (2) with $-(i/2k)$ and then adding $(\partial\Psi/\partial z)$ on each side, we obtain

$$-\frac{i}{2k}\frac{\partial^2\Psi}{\partial z^2}+\frac{\partial\Psi}{\partial z}=\frac{iP}{2k}\Psi+\frac{\partial\Psi}{\partial z}, \tag{3}$$

where $k=k_0n_{\mathrm{ref}}$; n_{ref} is the reference refractive index.

We may formally rewrite Eq. (3) in the form

$$\frac{\partial\Psi}{\partial z}=\frac{\dfrac{iP}{2k}+\dfrac{\partial}{\partial z}}{1-\dfrac{i}{2k}\dfrac{\partial}{\partial z}}\Psi. \tag{4}$$

Equation (4) suggests the recurrence relation

$$\left.\frac{\partial}{\partial z}\right|_{n+1}=\frac{\dfrac{iP}{2k}+\left.\dfrac{\partial}{\partial z}\right|_n}{1-\dfrac{i}{2k}\left.\dfrac{\partial}{\partial z}\right|_n}. \tag{5}$$

By using the initial value of $(\partial/\partial z)|_0=0$, this gives us the Padé$(m,n)$-approximant-based WA beam propagation formula as follows:

$$\frac{\partial \Psi}{\partial z} \approx ik \frac{N(m)}{D(n)} \Psi, \tag{6}$$

where $N(m)$ and $D(n)$ are polynomials in $X=(P/k^2)$. The most useful low-order Padé approximant operators are shown in Table 1.

B. Analytical Assessment of WA-BPM

If Eq. (4) is compared to a formal solution of Eq. (2) written in the well-known form

$$\frac{\partial \Psi}{\partial z} = i\sqrt{P}\Psi = ik\sqrt{X}\Psi, \tag{7}$$

we obtain the approximation formula

$$\sqrt{X} \approx \frac{N(m)}{D(n)}. \tag{8}$$

Since the operator X has a real spectrum, it is useful to consider the approximation of $\sqrt{X}$ by the Padé approximant propagation operators. Figure 1 shows the absolute values of $\sqrt{X}$ and the most useful low-order Padé(m,n) approximant operators mentioned as $KP(m,n)$ with respect to X. It is shown that the approximations $KP(m,n)$ are a good fit to the exact solution of the scalar Helmholtz equation. Furthermore, it is clearly seen that the higher the order of the approximation $KP(m,n)$ is, the more accurate the approximation to the Helmholtz propagator is. Figure 1 also shows the approximate approaches to the propagation operator using a traditional Padé approximant [10,11] and the Hadley(m,n)-approximant-based approach [3]. It is obvious that our resulting method allows one a more accurate approximation to the Helmholtz propagator in a wide range of operator X than the previous approaches. However, if the denominator of the approximation $KP(m,n)$ formula approaches zero, its absolute value approaches ∞ as clearly seen in Fig. 2. Physically, our resulting method correctly models waves propagating in the propagating region, where $X \geqslant 0$, whereas it incorrectly models waves propagating in the evanescent region, where $X<0$. To circumvent this problem, we employ the rotation technique of the square-root operator in the complex plane to address the evanescent waves proposed by Milinazzo *et al.* [12]. From Fig. 2, it is clearly shown that the rotated $KP(1,1)$ could give the evanescent wave the desire damping and allow one a good approximation to the true Helmholtz equation.

Table 1. Most Useful Low-Order Padé Approximants for Helmholtz Propagator in Terms of the Operator $X=P/k^2$

Order	Expression
(1,0)	$\frac{X}{2}$
(1,1)	$\frac{X}{1+\frac{X}{4}}$
(2,2)	$\frac{2X+\frac{X^2}{2}}{1+\frac{3X}{2}+\frac{X^2}{16}}$
(3,3)	$\frac{3X+\frac{5X^2}{2}+\frac{3X^3}{16}}{1+\frac{15X}{4}+\frac{15X^2}{16}+\frac{X^3}{64}}$

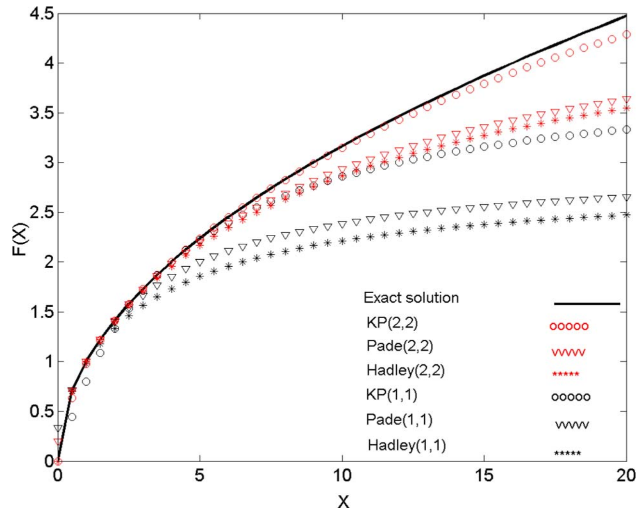

Fig. 1. (Color online) Absolute values of $\sqrt{X}$ (solid curve), the most useful low-order traditional Padé approximants, Hadley(m,n), and $KP(m,n)$ of $\sqrt{X}$ (dotted curves).

C. Numerical Implementation of WA-BPM

One of the most commonly used techniques to numerically deal with Eq. (6) is the finite difference method [3]. Finite difference equations may be derived from Eq. (6) by clearing the denominator and centering with respect to z in the usual way,

$$D(\Psi^{n+1} - \Psi^{n}) = \frac{ik\Delta z}{2} N(\Psi^{n+1} + \Psi^{n}). \tag{9}$$

Equation (9) can be solved effectively by the multistep method whereby each component step is treated by the

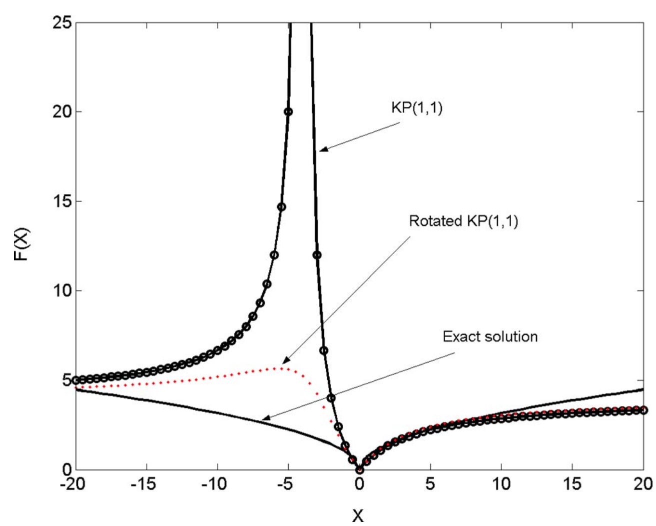

Fig. 2. (Color online) Absolute values of $\sqrt{X}$ (solid curve), the first-order (solid curve with circles), and rotated (dotted curve) $KP(1,1)$ approximant of $\sqrt{X}$.

traditional direct matrix inversion for two-dimensional (2D) problems [13]. However, for large three-dimensional (3D) problems requiring the frequently matrix inversion during a propagation direction it is a numerically intensive task. Recently, we reported the approach solving these problems effectively and accurately by using the new complex Jacobi iterative (CJI) method [4]. The utility of the CJI technique depends mostly upon its execution speed dominated by the amount of effective absorption (or medium loss). If the medium loss is high, the convergence rate is thus fast.

For WA-BPMs based on the Hadley(1,1) approximant, the propagation equation is given by

$$(1+\xi_{\mathrm{Hadley}}P_{\mathrm{Hadley}})\Psi^{n+1}=(1+\xi^{*}_{\mathrm{Hadley}}P_{\mathrm{Hadley}})\Psi^{n}, \quad (10)$$

where $P_{\mathrm{Hadley}}=\nabla_{\perp}^{2}+k_0^2(n^2-n_{\mathrm{ref}}^2)=(\partial^2/\partial x^2)+(\partial^2/\partial y^2)+k_0^2(n^2-n_{\mathrm{ref}}^2)$, $\xi_{\mathrm{Hadley}}=(1/4k^2)-(i\Delta z/4k)$, $\xi^{*}_{\mathrm{Hadley}}$ is the complex conjugate of ξ_{Hadley}, and Δz is the propagation step size. For those based on the $KP(1,1)$ approximant, the propagation equation is given by

$$(1+\xi_{KP}P_{KP})\Psi^{n+1}=(1+\xi^{*}_{KP}P_{KP})\Psi^{n}, \quad (11)$$

where $P_{KP}=\nabla_{\perp}^{2}+k_0^2n^2=(\partial^2/\partial x^2)+(\partial^2/\partial y^2)+k_0^2n^2$, $\xi_{KP}=(1/4k^2)-(i\Delta z/2k)$, and ξ^{*}_{KP} is the complex conjugate of ξ_{KP}.

If the CJI technique is employed to solve Eqs. (10) and (11), their medium losses are determined by the imaginary part of $1/\xi_{\mathrm{Hadley}}$ and $1/\xi_{KP}$, respectively [4]. The amount of the medium loss of the WA beam propagation equation based on Hadley(1,1) and $KP(1,1)$ approximants with respect to the propagation step size Δz at the wavelength $\lambda=0.633$ μm and the reference refractive index $n_{\mathrm{ref}}=3.3$ is depicted in Fig. 3. It is seen that for $\Delta z<0.0216$ μm the medium loss (and thus the convergence rate) of the WA propagation equation based on $KP(1,1)$ is higher than that of Hadley(1,1). Therefore, the loss is high for a typical choice of $k\Delta z$. This is a condition that favors a more rapid convergence for a $KP(1,1)$ approximant-based WA-BPM using CJI than that of Hadley(1,1).

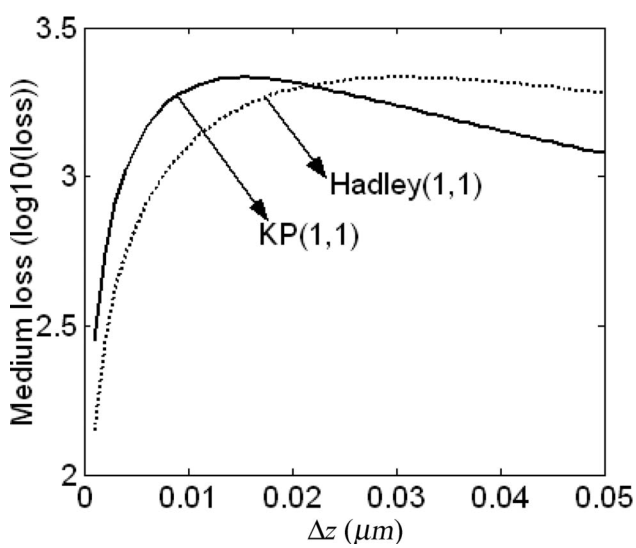

Fig. 3. Medium loss of WA beam propagation using CJI based on Hadley(1,1) (dotted curve) and $KP(1,1)$ approximant (solid curve).

3. APPLICATION

To prove the applicability and the accuracy of this method, we now employ it to study 2D and 3D optical waveguide problems whereby the WA beam propagation is needed and compared with those obtained by the existing approach. For the 2D case, we consider a Y-junction waveguide. The parameters needed for the calculations are the same as in [14]. The fundamental mode for the slab of width $w=0.2$ μm after propagating through 21 μm at wavelength $\lambda=0.633$ μm is depicted in Fig. 4. For a small propagation step size $\Delta z=0.01$ μm due to the high effective loss in the propagation medium the CJI technique for WA-BPM based on the $KP(1,1)$ approximant performed the propagation in 1047 s whereas that of the Hadley(1,1) approximant required 1552 s.

For the 3D case, we consider the guided-mode propagation in the Y-branch rib waveguide [4]. The initial rib waveguide is split into two 10° tilted waveguides. The fundamental mode of the ridge waveguide of width $w=2$ μm for the polarization TE mode at 0.633-μm in wavelength is used as the excited field at $z=0$. The field pattern at $z=3(\mu\text{m})$ calculated by Hadley's and our approach is depicted in Fig. 5. For a propagation step size $\Delta z=0.1$ μm, the resulting time of CJI for the Hadley(1,1)-approximant-based WA-BPM is 42 s while runtime for that of the $KP(1,1)$-approximant-based one is 61 s. It is seen that the CJI for the $KP(1,1)$-approximant-based WA-BPM is faster for a small propagation step size and slower for a large one than that of the Hadley(1,1)-approximant-based WA-BPM. This is due to the amount of effective absorption as shown in Section 2. As shown in Figs. 4 and 5, for modeling these structures the improvement of our approach compared to Hadley's one in terms

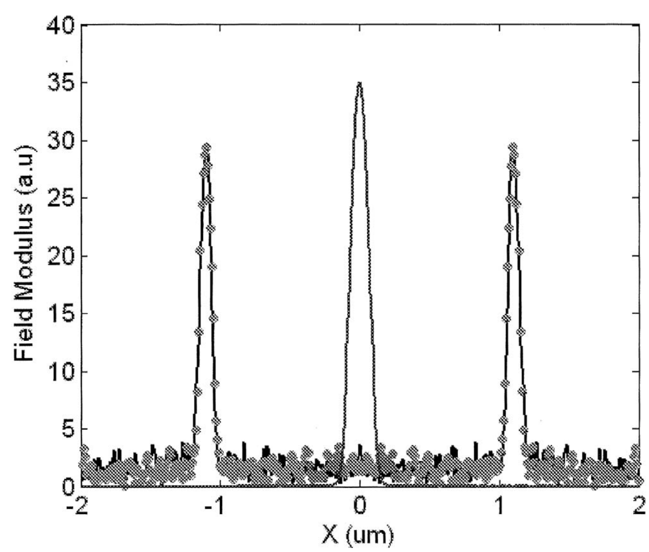

Fig. 4. Input beam at $z=0$ (solid curve that peaks in center) and output beam at $z=21$ μm in a 2D Y-branch rib waveguide calculated by WA-BPM based on $KP(1,1)$ (solid curve that peaks on left and right) and Hadley(1,1) (circles).

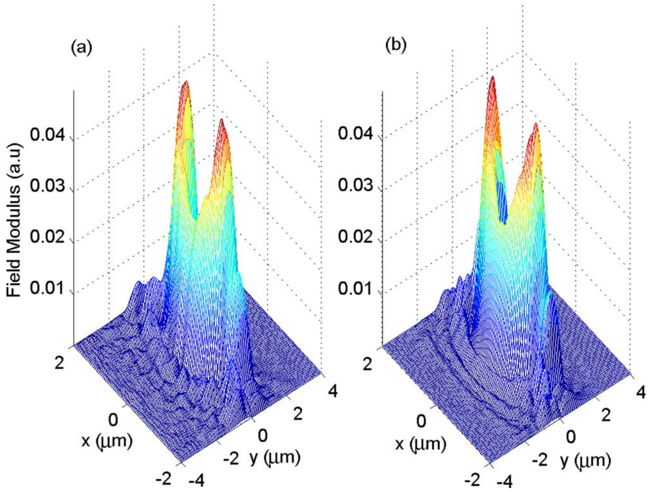

Fig. 5. (Color online) Magnitude of TE fundamental mode after propagating 3 μm in a 3D Y-branch rib waveguide calculated by WA-BPM based on (a) $KP(1,1)$ and (b) Hadley(1,1).

of accuracy is not too much. However, in terms of execution speed our approach can show a significant improvement. This is attributed to how high the medium loss (and thus the convergent rate of CJI dominated by a typical choice of $k\Delta z$) is.

4. CONCLUSIONS

In this paper, we have derived a new WA-BPM based on Padé approximant operators. The resulting approach allows one more accurate approximations to the true Helmholtz equation than the previous Padé-approximant-based approaches in a wide range of operator X. In addition, for a typical choice of $k\Delta z$, the convergence rate of CJI for WA-BPM based on our new approach is faster than that of Hadley's one. Furthermore, in contrast to existing methods, no slowly varying field approximations are assumed so that WA formulations are completely general.

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