Influence of the Carrier Density Dependence of the Absorption on the Harmonic Distortion in Semiconductor Lasers

Geert Morthier

Abstract—**It is shown that the carrier density dependence of the absorption can result in a significant increase of the harmonic distortion caused by gain suppression or by spatial hole burning in DFB-lasers. This carrier density dependence of the absorption is often neglected in theoretical studies, but we found that it may result in a dominant contribution to the second-order distortion in the AM-response, especially at high output powers. The distortion then mainly depends on material parameters such as differential gain, differential absorption and the gain suppression coefficient.**

I. Introduction

Analog optical communication systems require single frequency laser diodes (e.g., DFB-lasers) with a low harmonic and intermodulation distortion. However, the theoretical and experimental investigation of the distortion has started only a few years ago. The influence of several nonlinearities such as gain suppression [1], longitudinal spatial hole burning [1]–[3], leakage currents [4] and the relaxation oscillations has been addressed, but the theoretically calculated distortion always seems to be lower than the experimentally measured distortion. In [3] an experimentally measured second-order distortion between −50 and −40 dB is reported for a bias power of 10 mW, a modulation frequency of 250 MHz and a modulation depth of 25%. Similar values for the experimentally measured second-order distortion are given in [4] for a modulation frequency of 50 MHz and a modulation depth of 10%. On the other hand, theoretical calculations taking into account leakage currents [4] or spatial hole burning and gain suppression [1] seem to give values for the second-order harmonic distortion which are 10 to 20 dB below the experimental values.

We will show that a much better agreement between simulation results and experimental values may be obtained if the carrier density dependence of the absorption is included. This dependence is usually neglected in numerical simulations or in a small signal analysis, yet it can easily result in, e.g., a 10 dB increase of the second-order harmonic distortion in the AM-response at an output power of 10 mW. The importance of this carrier density dependence of the absorption has been recognized earlier by Lacourse *et al.* [5] for the explanation of the decrease in differential quantum efficiency with current.

Manuscript received September 26, 1991; revised April 27, 1992.
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IEEE Log Number 9205527.

We derived explicit formula for the second- and third-order harmonic distortions (H_2 and H_3) caused by the gain suppression and the carrier density dependence of the absorption. This shows the importance of several material parameters, but also of the photon distribution inside the laser cavity. Typical numerical values are used to obtain estimates of the second- and third-order distortion in MQW Fabry–Perot lasers. We finally also consider $\lambda/4$- shifted DFB-lasers to study the influence of both spatial hole burning and gain suppression.

II. Theory

We can derive analytical expressions for H_2 and H_3 from the single-mode rate equations for the number of photons I in the laser cavity and for the average carrier density N in the active layer

$$\frac{dI}{dt} = [G - \gamma]I \tag{1a}$$

$$\frac{dN}{dt} = \frac{J}{qd} - \left(\frac{N}{\tau} + BN^2 + CN^3\right) - \frac{GI}{V} \tag{1b}$$

with $G = \Gamma g v_g$ being the modal gain and γ the modal loss. J is the current density injected into the active layer with thickness d and volume V. Γ is the confinement factor of the active layer and v_g is the group velocity. After taking gain suppression (caused by, e.g., spectral hole burning) into account, we can express the gain as

$$G = A(N - N_t)(1 - \xi I), A = \Gamma v_g \frac{dg}{dN}. \tag{2}$$

The loss γ consists of absorption loss γ_a and facet loss γ_m, with

$$\gamma_a = \gamma_{a,0} + \gamma_{a,1}N = v_g(\alpha_{a,0} + \alpha_{a,1}N)$$
$$\alpha_{a,1} = \Gamma\frac{d\alpha_{\text{act}}}{dN} \tag{3}$$

with α_{act} the absorption in the active layer. α_{act} mainly consists of intervalence band absorption, in which a direct transition of a hole from the heavy or light hole band to the split-off band occurs. The probability for this transition depends on the occupation of the valence band and hence on the hole or electron density.

From (1)–(3), it follows that any change of the carrier density with increasing power level causes a proportional change of the internal absorption. This changed absorption in turn will require additional changes of the carrier density in order to keep the gain equal to the loss. The carrier density dependence of both gain and loss therefore has a large influence on the final change of the carrier density (and hence of the absorption). Eventually, the variation of the absorption implies a variation of the external efficiency, which implies harmonic distortion. It must be emphasized however that the carrier density dependence of the absorption itself is not the cause of the distortion. As long as the carrier density remains pinned above threshold (e.g., in the absence of gain suppression, leakage currents and spatial hole burning), a constant absorption and efficiency is obtained. The carrier density dependence of the absorption "amplifies" the effect of real nonlinearities such as gain suppression.

We will further only consider low modulation frequencies, i.e., modulation frequencies for which the influence of the relaxation oscillations on the distortion can be neglected. The contribution of the relaxation oscillations increases with 40 dB per decade and the actual frequency range where it can be neglected depends of course on the magnitude of the other contributions to the distortion. The low frequency approximation, which is based on the neglectance of the time derivatives in (1), is usually valid for modulation frequencies below a few hundred megahertz. Under modulation of the current J, we can write for a modulation frequency $\omega/2\pi$

$$J = J_0 + J_1 \cos(\omega t),$$
$$N = \sum_{k=0}^{\infty} N_k \cos(k\omega t),$$
$$I = \sum_{k=0}^{\infty} I_k \cos(k\omega t). \tag{4}$$

The value of H_2 and H_3 can be obtained as I_2/I_1, resp. I_3/I_1 after substitution of (4) in (1) and the application of a small signal analysis. We first neglect spatial hole burning. As has been outlined in [1], longitudinal spatial hole burning leads, via a longitudinal variation of the carrier density and hence a longitudinal variation of the gain and the refractive index, to a dependence of the facet losses γ_m on the power level. Here, we consider γ_m as a constant. One then finds for H_2 and H_3:

$$\frac{P_2}{P_1} = \frac{I_2}{I_1} = -\frac{\gamma_{a,1}}{A - \gamma_{a,1}}\left(1 + \frac{\xi I_0 A}{A' - \gamma_{a,1}}\right)\frac{\xi I_1}{2(1 - \xi I_0)}$$
$$A' = A(1 - \xi I_0) \tag{5a}$$

$$\frac{P_3}{P_1} = \frac{I_3}{I_1} = \frac{P_2}{P_1}\left\{2\frac{P_2}{P_1} + \frac{\xi I_1 A}{2(A' - \gamma_{a,1})}\right\}. \tag{5b}$$

The influence of the different material parameters can be seen immediately from (5). Furthermore, since system design usually requires a certain amount of modulated output power, one has to take into account the relation between modulation of the photon number I_1 and modulation of the output power P_1. We will address this furtheron.

To derive typical values for the contributions (5) to the harmonic distortion, we first consider a MQW Fabry–Perot laser with cleaved facets and with other parameters as in Table I [6]. We assume a static output power (through one facet) of 10 mW and an optical modulation depth of 30%. In addition, it can be noticed that the average internal power level in a Fabry–Perot laser with cleaved facets is about twice ($(1 + R)/(1 - R)$, with R the power reflectivity) the output power. The quantities ξ, I_0 and I_1 are then easily obtained as:

TABLE I

Quantity	MQW
Length L (μm)	400
Active layer thickness d (μm)	0.054
Stripe width w (μm)	1.5
Effective refractive index n_e	3.188
Group index n_g	4
Confinement factor Γ	0.088
Carrier lifetime τ (ns.)	5
Bimolecular coefficient B (cm^3/s)	$2\ 10^{-10}$
Auger coefficient C (cm^6/s)	10^{-28}
Linewidth enhancement factor α	3.5
Differential gain dg/dN (cm^2)	$7\ 10^{-16}$
Carrier density at transparancy N_t (cm^{-3})	$8\ 10^{17}$
Differential loss $d\alpha_{act}/dN$ (cm^2)	$3\ 10^{-16}$
Gain suppression ε (cm^3)	$4\ 10^{-17}$
Internal loss $\alpha_{a,0}$ (cm^{-1})	10

$$\xi = \frac{\Gamma\varepsilon}{\omega d L} = 1.0864\,10^{-7},$$
$$I_0 = \frac{2P_0 L}{E_g v_g} = 8.334\,10^5,$$
$$I_1 = 0.3\,I_0 = 2.5\,10^5$$

and substitution of these numerical values and the values given in Table I, yields for the second- and third-order harmonic distortion:

$$\frac{P_2}{P_1} = -\,0.0132 \text{ and } H_2 = 20\log_{10}\left(\left|\frac{P_2}{P_1}\right|\right) = -37.6\,\text{dB}$$
$$\frac{P_3}{P_1} = -\,2.1\,10^{-5} \text{ and } H_3 = 20\log_{10}\left(\left|\frac{P_3}{P_1}\right|\right) = -93.4\,\text{dB}.$$

We have performed numerical simulations of the harmonic distortion for the same Fabry–Perot laser, taking also into account the influence of the spontaneous carrier recombination (e.g., via traps, bimolecular and Auger recombination) as well as the (in Fabry–Perot lasers) weak spatial hole burning. At a modulation frequency of 50 MHz, we obtained the values $H_2 = -37.2$ dB and $H_3 = -93.8$ dB. The agreement with the analytical values indicates that (5) forms dominant contributions to the distortion, at least in Fabry–Perot lasers with cleaved facets.

The obtained value for the distortion depends strongly on the choice for dg/dN, $d\alpha_{\text{act}}/dN$ and ε. It can be mentioned that reported values of dg/dN for MQW material range from 2.7 $10^{-16}\,\text{cm}^2$ to $10^{-15}\,\text{cm}^2$, [10], [11], while the reported values for ε range from 4. $10^{-17}\,\text{cm}^3$ to 7.6 $10^{-17}\,\text{cm}^3$ [6], [12]. Values for $d\alpha_{\text{act}}/dN$ are rarely reported, [13] gives the value 0.8 $10^{-16}\,\text{cm}^2$ for wells of 5 nm thickness. Taking the largest reported value for dg/dN and the smallest reported values for ε and $d\alpha_{\text{act}}/dN$ reduces the value of (5a) to -56 dB, a value that is considerably smaller but that is still comparable

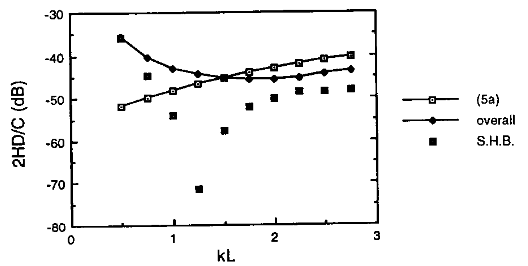

Fig. 1. Second-order harmonic distortion for AR-coated $\lambda/4$-shifted lasers at an output power of 10 mW and for $m = 30\%$.

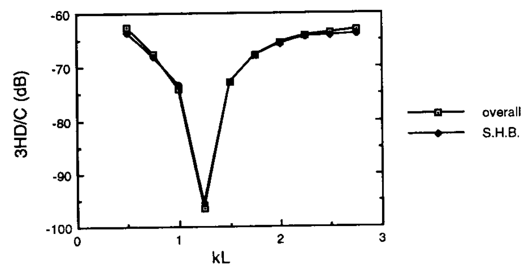

Fig. 2. Third-order harmonic distortion for AR-coated $\lambda/4$-shifted lasers at an output power of 10 mW and for $m = 30\%$.

to the second-order distortion caused by other effects. This best case is only obtained for extreme values of the reported material parameters and the choice of other parameters leads to a larger distortion. For a given choice of material parameters however, the formula (5a) and (5b) allow an easy calculation of the distortion caused by the carrier density dependence of the absorption.

III. Numerical Simulations for DFB Lasers

For MQW DFB-lasers with cleaved facets, the contribution (5a) still has a dominant influence on the second-order distortion, but the contribution (5b) is no longer dominant in DFB-lasers. Indeed, for typical DFB-lasers with cleaved facets, we have found that the spatial hole burning contribution to H_2 is typically less than -60 dB, while the contribution (5a) is usually more than -37 dB due to the power concentration in the middle of the laser in DFB-lasers. The spatial hole burning contribution to H_3 on the other hand is typically of the same order of magnitude as the contribution (5b).

The contribution (5a) can be reduced significantly by applying an AR-coating on one or both facets in order to reduce the photon number corresponding with a certain output power. An even larger reduction can be obtained with, e.g., AR-coated $\lambda/4$-shifted lasers with low κL-value, where the power is concentrated near the laser ends, which gives a small I_1/P_1 ratio. The value of H_3 given by (5b) becomes very small in this case and therefore the value of H_3 will be dominated by spatial hole burning or changes in the spontaneous carrier recombination.

Numerical results for 400-μm-long $\lambda/4$-shifted lasers are presented in Figs. 1 and 2, for the second, resp. third-order harmonic distortion. The lasers have been biased at 10-mW output power and an optical modulation depth m of 30% is used. The results have been calculated with the longitudinal laser diode model CLADISS [1], [7]. We will concentrate on the second-order distortion. Fig. 1 shows the contribution given by (5a) and the spatial hole burning (S.H.B.) contribution separately. One can see that the contribution (5a) is indeed much lower than in lasers with cleaved facets due to the smaller photon number that corresponds with the output power of 10 mW. This contribution increases from -52 dB for $\kappa L = 0.5$ to -40 dB for $\kappa L = 2.5$.

In spite of the very low value for the contribution (5a) at low κL-values, one can see that the overall distortion for these values is above -40 dB and that the overall distortion reaches a minimum for κL-values of about 1.75–2. This can be explained by taking the spatial hole burning contribution into account. This last contribution reaches a minimum for $\kappa L = 1.25$ and it has a phase π for smaller κL-values and a phase 0 (opposite to the contribution (5a)) for larger κL-values. Hence, for κL less than 1.25, the overall distortion is given by the sum of the contribution (5a) and the contribution from spatial hole burning. This contribution is also relatively large for $\kappa L = 0.5$–0.75. For κL-values above 1.25, the overall distortion is given by the difference between both contributions. Since the contribution from spatial hole burning is smaller than the contribution (5a), it follows that the overall distortion is a little smaller than the value given by (5a). It can thereby be remarked that the contribution from spatial hole burning doesn't seem to increase a lot with increasing κL-values for high κL-values, although one expects an increased spatial hole burning. The reason is that the spatial hole burning contribution is practically proportional with the absorption loss (see [1]), which decreases with increasing κL-value due to a decrease of the threshold carrier density and the carrier density dependence of the loss.

From Fig. 2, it follows immediately that the third-order distortion is practically determined by spatial hole burning, since taking gain suppression into account does not result in a significant change of H_3.

IV. Conclusion

It has been shown that a major contribution to the harmonic distortion in semiconductor laser diodes may have its origin in the carrier density dependence of the absorption and in gain suppression. Analytical formulas which give the distortion as a function of material parameters and of the average internal photon density have been presented. Numerical values for MQW lasers have been given.

The effect is particularly important at high bias powers, since the contribution to the second-order distortion described here increases proportional with the bias power. At low bias powers, other contributions to the distortion (such as spatial hole burning, which weakens with increasing bias power) may dominate the contribution discussed above.

The formulas indicate that a reduction of this particular contribution to the distortion can be obtained with lasers with a low ratio of photon number to output power. To this end, we investigated AR-coated $\lambda/4$- shifted lasers where, especially for low κL-values, the ratio of photon number to output power is low. However, it seems that the reduction of the distortion is not obtained due to a large spatial hole burning contribution [1] in this case. In lasers with weaker spatial hole burning, the application of an AR-coating on one or both facets may well lead to a reduced harmonic distortion.

ACKNOWLEDGMENT

The author is indebted to R. Baets (Univ. Gent), to F. Brillouet, J. Beylat, H. Bissessur (Alcatel Alsthom Recherche), and to U. Cebulla, H. Haisch (Alcatel SEL) for numerous discussions and to the management of Alcatel for funding this work and for permission to publish.

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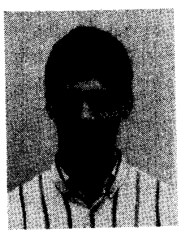

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